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# Strength predictions of clear wood at multiple scales using numerical limit analysis approaches

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ABSTRACT

This work aims at a new approach for understanding failure mechanisms and predicting wood strengths, which are strongly influenced by the complex hierarchical material system of wood. Thus, a mechanical concept, where different microstructural characteristics are incorporated, appears to be necessary, based on the division of wood into meaningful scales of observation. At each scale, effective strength properties are to be determined and a multiscale approach needs to be applied, for which conventional numerical methods appear to be inefficient. In this work, numerical limit analysis approaches are further developed and applied for the first time to wood, complementing conventional methods successfully at certain scales of observation in a multiscale 'damage' approach.

Limit analysis belongs to the group of direct plastic analysis methods, focusing exclusively on the time instant of structural collapse, and delivering the ultimate strength. Compared with conventional numerical approaches that have previously been applied to wood, limit analysis approaches are much more stable and efficient.

In this work, orthotropic failure criteria and periodic boundary conditions are implemented into both lower bound and upper bound numerical limit analysis formulations. As numerical results, effective failure surfaces are obtained at both annual ring scale and clear wood scale. A validation at clear wood scale indicates that this new approach is very promising.

## 1. Introduction

In recent years, wood, as a building material, has undergone a revival. This can be attributed to its excellent mechanical and physical properties on one hand and the fact that it is an environmentally sustainable material with a pleasant appearance on the other hand. Also, due to continuous extensions and improvements in building codes throughout Europe, allowing higher and more complex timber constructions, its share of the building market is constantly increasing and the volume of consumption is experiencing enormous growth rates. In the course of this, however, demands on timber constructions are increasing constantly. In order to meet these demands and allow the use of wood in complex applications, prediction tools for the mechanical performance of wood are gaining importance. A wider repertoire of advanced prediction tools should facilitate a better utilisation of wood and wood-based products, increasing their competitiveness compared with other building materials. Especially for predicting the ultimate strength of wood, very few reliable and promising methods exist so far. A brief overview of some existing methods for predicting/modelling structural failure at the wooden board level is given in the following.

### 1.1. Prediction tools for the ultimate strength of wood

The first group of approaches avoids the direct description of failure mechanisms and instead uses so-called *mean stress concepts* [49], where averaged stresses over a finite small area are assumed to indicate failure. These areas can be adjusted to typical features of wood, such as structural characteristics of wood fibres [1]. Serrano and Gustafsson [62], Sjödin and Serrano [64] and Sjödin et al. [65] applied this approach in combination with findings of linear elastic fracture mechanics. They investigated single and multiple dowel connections, where the size of the finite area was governed by the fracture properties of the material. The suitability of different area definitions, over which the stresses are averaged, and also the efficiency of various failure criteria, were compared by Guindos [20]. Lukacevic and Füssl [32] presented a physically-based structural failure criterion, where it was assumed that in wooden boards with

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knots, global failure can be related to a stress transfer mechanism, which is identifiable by evaluating averaged stress fields in the vicinity of knots. Either way, such models are mostly dependent on empirical parameters and the true failure mechanisms cannot be identified. This can be overcome by directly modelling failure processes.

The most sophisticated approaches for this are based on multi-surface plasticity/failure criteria, as described in Mackenzie-Helnwein et al. [41] and Schmidt and Kaliske [58]. Thereby, orthotropic yield surfaces are defined describing the onset of plastic deformation, whereas failure surfaces indicate stress states where cracks are to be expected. Cracking is normally modelled with so-called cohesive elements, including an anisotropic traction separation law for wood. Applications of this model by Schmidt and Kaliske [59,60] show promising results regarding the estimation of load carrying behaviour. Danielsson and Gustafsson [7] also used a cohesive zone model based on plasticity theory for parametric studies of a glued laminated timber beam with a hole.

These approaches work well for a homogeneous fibre orientation and if the location of the crack path is known in advance. The weak point of these methods is the influence on the failure mechanism by specifying the crack direction. As wood is a naturally grown material, usually complex stress states prevail, especially in the presence of knots and other defects, meaning that such an approach is difficult to apply. In such cases additional strategies are required, like the approach in Jenkel and Kaliske [28], where cohesive interface elements are aligned with predetermined fibre directions around knots.

In recent years, Lukacevic and Füssl [32,33] and Lukacevic et al. [34,36] have established the basis for a crack initiation and propagation criterion in the framework of the extended finite element method (XFEM), which has been implemented into a numerical simulation tool for wooden boards. The implementation of such an approach poses two questions: (i) which stress states cause the initiation of a crack? and (ii) what is the corresponding crack direction at the wooden board scale? These questions can only be answered by looking deeper into the microscopic hierarchical structure of wood, and taking several structural features at different length scales into account. Therefore, to obtain reliable failure surfaces and reliable crack directions at the wooden board scale, a multiscale ‘damage’ approach is pursued. For such an approach, failure mechanisms at *n* different length scales of wood need to be analysed numerically. Doing this exclusively by applying the concept of multi-surface plasticity/failure criteria in combination with XFEM, at each length scale, leads to very high computational cost and probably to an unnecessarily high complexity of the overall model.

For this reason, an additional numerical method is to be introduced, namely *numerical limit analysis*. This method, a so-called ‘direct method’, focuses exclusively on the time instant of failure, and delivers lower and upper bounds for the ultimate strength of the considered material structure. Compared with conventional numerical approaches, where the complete load history has to be considered and, in order to predict correct failure mechanisms, proper regularisation techniques must be used, the limit analysis approach is much more stable and efficient. Moreover, it leads to rigorous bounds on the material strength and, thus, gives a reliable error measure for the prediction. Thus, this method can serve as a useful tool for complementing more complex numerical step-by-step approaches by, for example, identifying critical failure regions in a preliminary simulation procedure, as also suggested in Füssl et al. [15] and Pisano et al. [54].

Of course, these advantages result from the strict limitations on which these formulations are based, including: (i) the associated plastic flow rule and (ii) the rigid and perfectly plastic (ductile) material behaviour. For wood, these two idealisations are not entirely correct, but the first can be considered as an appropriate assumption, which is made due to a lack of information about the non-associativity, and the second does not exclude that good strength predictions are also possible for strain-softening structures. In Denton and Morley [8] it is stated: “A structure does not need to exhibit perfect plasticity for the theoretical plastic collapse load based on the peak yield stress of each component to be approached closely. Rather, it is necessary that, at the point when a collapse mechanism forms under a particular loading, all those regions within the structure which are undergoing straining lie very close to the peak yield stress which they can achieve.” Wood definitely has the ductile potential to ‘activate’ the strength of many points along a potential crack surface before brittle failure occurs. Nevertheless, one might argue that this approach, mainly evolved from and applied in fields dealing with very homogeneous (man-made) materials like steel, is not suitable for application to wood, where failure is often induced by the largest defect (such as knots at the wooden board scale, or cell wall imperfections at an observation scale below). With regard to this, it might be mentioned that the prediction quality of concepts addressing structural failure of a highly heterogeneous material depends heavily on knowledge about the local strength reduction due to defects. Thus, tools which are capable of analysing this influence for a huge number of defect variations within an acceptable timeframe, like numerical limit analysis, might be very useful for fracture models at the macroscopic scale. For this reason, the numerical limit analysis approach seems to be an appropriate method to make a comprehensive multiscale ‘damage’ framework for wood worth pursuing.

### 1.2. Numerical limit analysis

Originally, the objective of limit analysis was the determination of the load bearing capacity of structures exhibiting elastoplastic material response. At collapse, the capacity of structures to store any additional external work as recoverable energy is lost. Thus, for a prescribed macroscopic velocity field and a prescribed macroscopic traction field on the boundary, defining the loading situation, limit analysis concentrates on the critical energy dissipation rate at failure of structures or, in this paper, of unit cells for microstructures. The problem may be stated as follows according to Ciria et al. [6]: *Find the kinematically admissible velocity field, which minimises the external energy over the set of all statically admissible stress fields, which maximise the internal dissipated energy*. Unfortunately, the resulting saddle-point problem can be solved exactly only for simple geometric and loading situations, and for simple material behaviour. For more complex situations, the plastic flow compatibility in the static lower bound principle and the plastic admissibility in the kinematic upper bound principle may be relaxed, providing lower and upper bounds for the load bearing capacity (effective strength) of structures.

The first complete formulations of the limit analysis theorems were established in the 1950s by Drucker et al. [9,10] and Hill [22], though analytical exact solutions (coincident lower and upper bounds) were limited to very simple problems. Thanks to the rapid evolution of computer technology and developments in mathematical programming, the finite element method (FEM) has proven to be a powerful tool for implementing the limit analysis approach, from simple two-dimensional problems to complicated three-dimensional applications. Therefore, increasing attention has been given to numerical limit analysis formulations within past decades.

Early implementations of limit analysis using the finite element method in conjunction with numerical optimisation were performed by Lysmer [40] for the lower bound problem, and by Anderheggen and Knöpfel [2] and Maier et al. [42] for the upper bound problem. In these works, linear three-noded triangular elements were used for discretisation and the resulting optimisation

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