ARTICLE IN PRESS

Computers and Structures xxx (2017) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

A Rayleigh-Ritz approach for postbuckling analysis of variable angle tow composite stiffened panels

V. Oliveri [a,*], A. Milazzo [b]

[a] Bernal Institute, School of Engineering, University of Limerick, V94 T9PX, Limerick, Ireland
[b] Dipartimento di Ingegneria Civile, Ambientale, Aerospaziale, dei Materiali, University of Palermo, Viale delle Scienze, Edificio 8, 90128 Palermo, Italy

ARTICLE INFO

Article history:
Received 17 May 2017
Accepted 17 October 2017
Available online xxxx

Keywords:
Variable angle tow composites
Composite stiffened plates
Postbuckling analysis
Rayleigh-Ritz method
First order shear deformation theory

ABSTRACT

A Rayleigh-Ritz solution approach for generally restrained multilayered variable angle tow stiffened plates in postbuckling regime is presented. The plate model is based on the first order shear deformation theory and accounts for geometrical nonlinearity through the von Kármán's assumptions. Stiffened plates are modelled as assembly of plate-like elements and penalty techniques are used to join the elements in the assembled structure and to apply the kinematical boundary conditions. General symmetric and unsymmetric stacking sequences are considered and Legendre orthogonal polynomials are employed to build the trial functions. A computer code was developed to implement the proposed approach and to establish its applicability and its features for investigating variable angle tow structures. The proposed solution is validated by comparison with literature and finite elements results. Original results are presented for postbuckling of variable angle tow stiffened plates showing the potentialities of the method.

1. Introduction

The high mechanical properties offered by fibre-reinforced laminates have made essential their application as components of advanced and lightweight structures. Widely employed in automotive, naval and aerospace applications, composite laminates are often designed as stiffened panels or thin-walled structures.

These structures are able to sustain mechanical loads well after the occurrence of buckling. Thus, especially for aerospace applications, the accurate analysis of the postbuckling regime of multilayered composite plates becomes relevant in the design to increase weight savings and to improve safety margins.

The introduction of variable angle tow (VAT) composites [1,2] provides new ways to design high performance composite structures. It redefines the tailoring concept by distributing lay-ups with certain fibre orientations across the planform of the plate. This improves the structural performances; for example, VAT composite plates undergoing compression loads showed an improvement up to 50% of buckling load over conventional straight fibre composites, motivating the attention to this new class of laminates [3].

It should be mentioned that the enhancement of composite structures performances can be also achieved by using functionally graded materials, especially when exposed to thermal environment. They are characterized by a continuous through-thickness variation of their composition and then of their mechanical properties, which can be tailored to get the desired behaviour (e.g. [4–13]).

Literature survey shows that the Rayleigh-Ritz method is one of the most successful approach to describe with adequate accuracy the buckling and postbuckling behaviour of composite plates and that it is suitable to be implemented with high computational efficiency. Rayleigh-Ritz solutions for static loading, free vibrations, buckling and postbuckling analysis of composite plates have been proposed. However, most of the proposed Rayleigh-Ritz solutions implement the classical laminated plate theory (CLPT) [14–16] that can be successfully applied for thin plates, still it neglects the transverse shear strains, which can play an important role in thick plates or low transverse shear stiffness structures. The first-order shear deformation theory (FSDT) appears adequate for the engineering analysis and design of thin to moderately thick composite laminates [16] and it appears appealing if compared to more sophisticated higher order plate theories, due to its simplicity and low computational costs. Focusing on FSDT modeling of plates solved by the Rayleigh-Ritz method, different kind of trial functions have been proposed, showing reliable results for static analysis [17,18], free vibrations [19–24], buckling [17,19] and postbuckling [25–27] of thin to moderately tick composite laminated plates. Adopting classical thin plate theory, the Rayleigh-Ritz method has

* Corresponding author.
E-mail addresses: vincenzo.oliveri@ul.ie (V. Oliveri), alberto.milazzo@unipa.it (A. Milazzo).

https://doi.org/10.1016/j.compstruc.2017.10.009
0045-7949/

been used also to analyze the buckling and postbuckling behaviour of stiffened plates [28–33] and shells [34,35]. As regard variable stiffness composite structures, starting from the pioneering works by Gürdal and Olmedo [2], attention has been devoted to VAT composites as demonstrated by the recent literature on the subject [36–46]. In the field of the buckling of VAT structures, few results are available for stiffened panels [47] and for modeling techniques employing FSDT [48] or higher order theories [49].

To the best of author's knowledge, the postbuckling behaviour of VAT composite plates with integrated VAT stiffeners remains still unexplored.

Recently, the authors presented a Rayleigh-Ritz approach for large deflection analysis of composite panels and thin-walled structures based on FSDT [50,51], demonstrating its ability in modeling the postbuckling behaviour. In the present study, this approach is extended to VAT stiffened plates and thin-walled structures. In particular, a Rayleigh-Ritz solution for generally restrained multilayered stiffened VAT panels in postbuckling regime is presented. Stiffened VAT plates are modelled as assembly of plate-like elements, over which a varying fibre orientation angle is considered. The plate modeling is based on the first order shear deformation theory and accounts for geometrical nonlinearity through the von Kármán's assumptions. Penalty techniques are used to enforce the displacements continuity of the multidomain assembled structure and to apply the kinematical boundary conditions. Legendre orthogonal polynomials are employed to approximate the displacement field. A computer code has been developed to implement the corresponding Rayleigh-Ritz solution for postbuckling analysis of stiffened composite VAT plates with general configurations and loadings. Validation and original results are finally presented. The aim of the paper is to verify the applicability of the approach based on the Rayleigh-Ritz method and domain decomposition to the study of complex VAT structures in postbuckling regime, ascertaining its accuracy, effectiveness, computational cost and potentialities.

2. Formulation

2.1. Modeling strategy and definitions

The proposed modeling strategy consists of the decomposition of the whole structure into plate-like subdomains, referred in the following as plates or elements. The first-order shear deformation theory is adopted in order to obtain the governing equations of each multilayered plate as a single separate entity. In turn, the whole thin-walled structure is assembled by enforcing the boundary conditions for each component; these are given by displacement continuity and traction equilibrium along the edges joining different elements and by the external load and kinematical constraint conditions.

Consider a thin-walled structure thought as assembly of N_P quadrilateral composite multilayered plates and let the superscript k inside angle bracket denotes quantities associated with the k-th element. Each plate can be kinematically constrained on the lateral boundary and it is subjected to domain and boundary loads as specified in the next Section. The k-th plate is referred to its own local cartesian coordinate system with the origin located at the plate center whose $x_3^{\langle k\rangle}$ axis is directed along the plate thickness whereas the $x_1^{\langle k\rangle}$ and $x_2^{\langle k\rangle}$ coordinates span the mid-plane. Let the domain occupied by the plate's mid-plane be denoted by $\Omega^{\langle k\rangle}$ being $\partial\Omega^{\langle k\rangle}$ its boundary. The whole structure is also referred to a global cartesian coordinate system whose axis are denoted by X_i. To deal with general shaped quadrilateral plates, a natural coordinate system $\xi^{\langle k\rangle}\eta^{\langle k\rangle}$ is introduced, which maps the plate mid-plane coordinates to the square domain $[-1,1]\times[-1,1]$. This allows describing the plate in-plane coordinates as

$$x_i^{\langle k\rangle} = \sum_{\alpha=1}^{4} g_\alpha(\xi^{\langle k\rangle},\eta^{\langle k\rangle})\, x_{i\alpha}^{\langle k\rangle} \qquad i = 1,2 \tag{1}$$

where $x_{i\alpha}^{\langle k\rangle}$ are the coordinates of the α-th vertex of the plate mid-plane along the i-th axis and

$$g_1 = \frac{1}{4}(1-\xi)(1-\eta) \tag{2a}$$

$$g_2 = \frac{1}{4}(1+\xi)(1-\eta) \tag{2b}$$

$$g_3 = \frac{1}{4}(1+\xi)(1+\eta) \tag{2c}$$

$$g_4 = \frac{1}{4}(1-\xi)(1+\eta) \tag{2d}$$

2.2. Isolated plate governing equations

Consider the k-th plate of the thin-walled structure as an isolated structural entity.

Referring to the local cartesian coordinate system $x_i^{\langle k\rangle}$ and employing the FSDT, the plate deformation is described by the displacement vector $\boldsymbol{d}^{\langle k\rangle} = \left\{ d_1^{\langle k\rangle} \quad d_2^{\langle k\rangle} \quad d_3^{\langle k\rangle} \right\}^T$, expressed as

$$\boldsymbol{d}^{\langle k\rangle} = \boldsymbol{u}^{\langle k\rangle} + \left(x_3^{\langle k\rangle} - \bar{x}_3^{\langle k\rangle}\right)\boldsymbol{L}\boldsymbol{\vartheta}^{\langle k\rangle} + \bar{\boldsymbol{w}}^{\langle k\rangle} \tag{3}$$

where

$$\boldsymbol{L} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

and $\bar{x}_3^{\langle k\rangle}$ is the offset that defines the so called modeling plane $x_3^{\langle k\rangle} = \bar{x}_3^{\langle k\rangle}$. In Eq. (3), the generalized displacement vectors are defined as $\boldsymbol{u}^{\langle k\rangle} = \left\{ u_1^{\langle k\rangle} \quad u_2^{\langle k\rangle} \quad u_3^{\langle k\rangle} \right\}^T$ and $\boldsymbol{\vartheta}^{\langle k\rangle} = \left\{ \vartheta_1^{\langle k\rangle} \quad \vartheta_2^{\langle k\rangle} \quad \vartheta_3^{\langle k\rangle} \right\}^T$ where $u_1^{\langle k\rangle} = u_1^{\langle k\rangle}(\xi,\eta), u_2^{\langle k\rangle} = u_2^{\langle k\rangle}(\xi,\eta)$ and $u_3^{\langle k\rangle} = u_3^{\langle k\rangle}(\xi,\eta)$ are the displacement components of the $\bar{x}_3$-plane points along the reference directions, whereas

$\vartheta_1^{\langle k\rangle} = \vartheta_1^{\langle k\rangle}(\xi,\eta)$ and $\vartheta_2^{\langle k\rangle} = \vartheta_2^{\langle k\rangle}(\xi,\eta)$ are the rotations of the transverse normal around the $x_2^{\langle k\rangle}$ and $x_1^{\langle k\rangle}$ axes, respectively, and $\vartheta_3^{\langle k\rangle} = \vartheta_3^{\langle k\rangle}(\xi,\eta)$ is the "drilling" rotation.

Note that the drilling rotation does not affect the plate deformation and its introduction relates to the enforcement of the interface conditions along the edges joining different plates [52] as described in Section 2.3. To consider initial imperfections of the structure, Eq. (3) accounts for the term $\bar{\boldsymbol{w}} = \left\{ 0 \quad 0 \quad \bar{w}^{\langle k\rangle} \right\}^T$ where $\bar{w}^{\langle k\rangle}$ is a prescribed initial transverse deflection of the plate modeling plane.

Adopting the total Lagrangian formulation and assuming moderately large displacements, the kinematical state is described by the Green's strains vector $\boldsymbol{e}^{\langle k\rangle}$ that is partitioned into the in-plane and out-of-plane components vectors denoted by the subscripts p and n, respectively:

$$\boldsymbol{e}^{\langle k\rangle} = \left\{ e_{11}^{\langle k\rangle} \quad e_{22}^{\langle k\rangle} \quad e_{12}^{\langle k\rangle} \quad | \quad e_{13}^{\langle k\rangle} \quad e_{23}^{\langle k\rangle} \quad e_{33}^{\langle k\rangle} \right\}^T = \begin{Bmatrix} \boldsymbol{e}_p^{\langle k\rangle} \\ \boldsymbol{e}_n^{\langle k\rangle} \end{Bmatrix} \tag{5}$$

Taking geometric nonlinearity into account trough the von Kármán's assumptions, the strain-displacement relations are given by

Please cite this article in press as: Oliveri V, Milazzo A. A Rayleigh-Ritz approach for postbuckling analysis of variable angle tow composite stiffened panels. Comput Struct (2017), https://doi.org/10.1016/j.compstruc.2017.10.009