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Axisymmetric semi-analytical finite elements for modelling waves in buried/submerged fluid-filled waveguides

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ABSTRACT

Efficient and accurate predictions of wave propagation are a vital component of wave-based non-destructive interrogation techniques. Although a variety of models are available in the literature, most of them are suited to a particular wave type or a specific frequency regime. In this paper we present a multi-wave model for wave propagation in axisymmetric fluid-filled waveguides, either buried or submerged in a fluid, based on the semi-analytical finite elements. The cross-section is discretised with high-order spectral elements to achieve high efficiency, and the singularities resulting from adopting a Lobatto scheme at the axis of symmetry are handled appropriately. The surrounding medium is modelled with a perfectly matched layer, and a practical rule of choice of its parameters, based only on the material properties and the geometry of the waveguide, is derived. To represent the fluid and the solid-fluid coupling, an acoustic SAFE element and appropriate coupling relationships are formulated. The model is validated against both numerical results from the literature and experiments, and the comparisons show very good agreement. Finally, an implementation of the method in Python is made available with this publication.

1. Introduction

Acoustic waves are perhaps the most common principle upon which modern non-destructive interrogation methods for buried/immersed pipes are developed [1,2]. Both wavespeed and attenuation depend on the properties of the pipe, its contents, and the surrounding medium, and can be used to identify these properties or evaluate their change over time. Moreover, when incident upon a discontinuity, either in the pipe or in the surrounding medium, the waves scatter, allowing for detection of defects or weakened supports. Finally, waves radiating from a pipe can be sensed at the ground surface and provide a basis upon which both the location and the condition of the pipe can be assessed remotely. An essential ingredient for all these techniques is a reliable model for wave propagation in buried/submerged pipes with fluid. Although there is a considerable number of publications dealing with either waves in fluid-filled pipes or with waves in embedded cylinders, relatively few works tackle the complete problem that includes both the pipe, the fluid and the surrounding medium.

For obvious historical reasons, analytical approaches were developed first. Dispersion curves and energy distributions for fluid-filled thin cylinders have been presented by Fuller and Fahy [3]. Pinnington and Briscoe derived low frequency approximations for both fluid-dominated and axial waves in free pipes. The effect of the surrounding fluid was investigated by e.g. Greenspon [4] and Sihna et al. [5]. The case of a solid medium restraining the pipe has been studied to some extent by Toki and Hakada [6] (in an earthquake engineering context) and by Jette and Parker [7].

The pipe, the surrounding medium and the contained fluid altogether were considered probably for the first time by Muggleton et al. [8–10] where both fluid-dominated and axial waves were studied based on a simplified interaction between the pipe and the soil/water. Subsequent refinements and extensions of that model allowing for inclusion of the shear coupling with lubricated contact [11], compact contact [12] and evaluating torsional waves [13] have also been published.

Despite the negligible computational cost and an immediate insight into the physics gained from closed-form expressions offered by the aforementioned models, they are often limited to a particular wave type and the low frequency range. A more versatile approach based on the global matrix method [14] was developed in the ultrasonic community and was successfully applied to the problem of embedded, fluid-filled cylinders [15,16], among others. The global matrix method originates from the description

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of the motion of the structure as a superposition of bulk waves propagating in each layer (the number of layers and materials can be arbitrary). The fundamental formulation is analytical, but requires a numerical solution in the form of root-tracing which often offers a fast and accurate solution. However, for complex structures, root-tracing may become both inefficient and unreliable (as it strongly depends on the initial guess). Moreover, developing a universal tracing algorithm applicable to all configurations is a rather cumbersome task.

The limitations of analytical models can be circumvented with numerical methods which solve the dispersion equation as an eigenvalue problem, such as the semi-analytical finite element (SAFE) method. The fundamental concept behind SAFE is that the cross-section is discretised using finite elements and spatially harmonic motion is assumed in the direction of propagation [17,18]. The governing equation is written as an eigenvalue problem which can easily be solved using any numerical package available. SAFE provides a stable and reliable solution at a cost of the finite element discretisation. However, the number of degrees of freedom is usually small for closed waveguides.

Modelling open (embedded) waveguides with SAFE raises several new challenges, of which the greatest is an efficient representation of the surrounding medium. Castaigns and Lowe [19] proposed the idea of an absorbing layer with material damping smoothly increasing away from the core. Their approach could be conveniently implemented in a commercial finite element package, but the size of the problem grew large for low wavespeed soils. Jia et al. [20] developed infinite elements that can be coupled to standard, solid SAFE elements and analysed the effect of soil on waves in hollow cylinders. Mazzotti et al. developed a hybrid approach based on SAFE and 2.5D boundary element method [21–23]. This idea yielded very promising results and was validated with experiments. However, coupling of two quite complex methods poses additional challenges for implementation.

An alternative is to use perfectly matched layers (PML) to represent the embedding medium. PMLs were first used in mid 1990s [24] and have since been successfully applied to a number of applications, particularly in the electromagnetic community. Including PMLs in numerical models for elastic waves is a relatively new idea. Treyssède et al. [25] developed a SAFE-PML formulation for plate like structures with a cubic polynomial chosen as a stretching function. Soon after, Nguyen et al. [26] presented an analogous formulation applied to three-dimensional embedded waveguides. However, for an acoustically slow surrounding medium, the size of the problem could grow large owing to short bulk wavelengths. As a remedy, Treyssède [27] applied spectral elements utilising high-order polynomials to enhance the efficiency of the solution. Also recently, Duan et al. [28] developed an axisymmetric SAFE-PML formulation and proposed the use of an exponential stretching function, particularly well suited to wave problems.

To enable SAFE-PML calculations to be configured rapidly, Zuo et al. [29] implemented this model in a commercially available software. The advantages of their approach are that one can benefit from the readily available meshing tools and can conduct simulations without writing any code. The authors also give guidelines on the choice of PML parameters. Some numerical examples for solid 1D and 2D cross-sections have also been presented. Despite, its user-friendliness, this approach still yields problems of considerable size.

A scaled boundary finite element method (SBFEM) is a good alternative to SAFE and has recently been applied to the problem of axisymmetric [30] and embedded waveguides [31–33]. SBFEM is a well-established numerical framework, particularly in earthquake engineering. Its key concept is to represent the computational domain using a discretised boundary and a scaling centre. For guided wave problems, the final form of the equations bears many similarities with those coming from SAFE, since in both cases the assumption of the harmonic variation of the displacement along the propagation direction is adopted. However, the key concept and the origin of SBFEM are inherently different from FEM. In the references mentioned above, the authors simplified the problem by representing the surrounding medium with a dashpot boundary condition. Although their approach is simple and elegant, it cannot provide accurate results when the contrast between acoustic impedances is low.

Despite the impressive developments in the field of modelling of elastic waves in recent decades, to the best of authors' knowledge, no complete and efficient numerical model for waves in fluid-filled embedded/submerged pipes has been published.[1] This paper proposes a SAFE formulation capable of representing the structural–acoustic coupling and benefiting from the circumferential periodicity assumption. Both the pipe and the surrounding medium can be composed of a number of layers of different, generally anisotropic materials. We use high-order spectral elements (SEs), as they are better suited to wave propagation problems than standard finite elements and allow for efficient simulations. The singularities arising from the axisymmetric assumption applied to SEs are accounted for appropriately. The surrounding medium, which can be either solid or fluid, is represented by a perfectly matched layer and practical guidelines for the choice of the parameters of the PML are derived. Our formulation is validated against published numerical results and experimental measurements showing very good agreement. Finally, an implementation of the proposed method in Python is made available with this publication.

2. Derivation of SAFE elements

We consider an axisymmetric, infinitely long and uniform waveguide, where r and θ are the cross-sectional coordinates and z is the direction of propagation. The waveguide can be composed of any number of layers, either solid of fluid. Owing to the axisymmetric assumption the cross-section is represented with monodimensional elements. A practical representation of such class of problems is an embedded/submerged fluid-filled pipe. The pipe scenario together with chosen labelling and conventions is shown in Fig. 1. SAFE formulations for all respective elements are presented in the following subsections.

2.1. Structural element

The structural (elastic) SAFE element is derived in a similar way to [18] but with slightly different conventions. We start from recalling the virtual work principle for deformable elastic bodies [35] which states that

$$\int_V \delta\bar{\mathbf{u}}^\top \rho \ddot{\bar{\mathbf{u}}}\, \mathrm{d}V + \int_V \delta\bar{\boldsymbol{\epsilon}}^\top \bar{\boldsymbol{\sigma}}\, \mathrm{d}V = \int_V \delta\bar{\mathbf{u}}^\top \bar{\mathbf{t}}\, \mathrm{d}V \tag{1}$$

where V is the volume occupied by the waveguide, $\bar{\mathbf{u}}$ is the displacement vector, ρ is the mass density, $\bar{\boldsymbol{\epsilon}}$ is the strain vector, $\bar{\boldsymbol{\sigma}}$ is the stress matrix, $\bar{\mathbf{t}}$ is the external traction and $\ddot{(\cdot)}$ symbol denotes double differentiation with respect to time. Our attention is focused on free wave propagation here, hence the right-hand side, i.e. the external traction, is set to zero.

The displacement and strain vectors are defined as

$$\begin{aligned} \bar{\mathbf{u}} &= [\,\bar{u}_r \quad \bar{u}_\theta \quad \bar{u}_z\,]^\top \\ \bar{\boldsymbol{\epsilon}} &= [\,\bar{\epsilon}_{rr} \quad \bar{\epsilon}_{\theta\theta} \quad \bar{\epsilon}_{zz} \quad \bar{\gamma}_{\theta z} \quad \bar{\gamma}_{zr} \quad \bar{\gamma}_{r\theta}\,]^\top \end{aligned} \tag{2}$$

[1] We acknowledge, that during the review process of this manuscript, Zuo and Fan [34] published an article on SAFE-PML modelling of structures immersed in a fluid, which bears some similarities with the approach presented in this paper.

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