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A three-dimensional plasticity-damage constitutive model for timber under cyclic loads

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ABSTRACT

The performance of timber structures is governed by the nonlinear response at their connections, where high deformation levels and stress concentrations are developed, particularly when subjected to load reversals. To date, no constitutive model for wood under cyclic load exists which is able to incorporate its most important failure modes while considering plastic deformations and cyclic stiffness and strength degradation simultaneously. This paper presents the formulation and implementation of a plasticity-damage model with these characteristics within a continuum mechanics approach. The theoretical framework of both plasticity and damage models is described, and a detailed derivation of the constitutive equations required for their computational implementation and coupling as well as the return mapping and iterative algorithms for their integration are presented. The damage evolution process is handled by two independent scalar variables for tension and compression. A general orthotropic plasticity yield surface with isotropic hardening is employed to incorporate timber plastic flow in compression. A closed-form expression for the plasticity-damage consistent tangent operator is derived. It is demonstrated that the proposed constitutive model captures all the key characteristics required for an accurate modelling of timber under large deformation levels until failure.

1. Introduction

The structural performance of timber structures is governed by the nonlinear response in the connection zones, where high deformation levels and stress concentrations are developed around the fasteners (nails, dowels or bolts). These zones are also susceptible to significant load-reversals during the service life of structures, especially during extreme scenarios such as earthquakes or strong winds. For this reason, performance-based assessment of timber structures requires the implementation of a material constitutive model capable of simulating the nonlinear behaviour of timber under large deformation levels and load reversals until failure.

Timber failure modes can broadly be defined as ductile failure due to compression stresses and brittle failure due to the combination of shear and tension stresses. Therefore, in addition to the anisotropy of timber, the numerical modelling of this material involves the challenge of reproducing completely different failure modes and nonlinear responses for tensile and compressive stress regimes. Although approximate phenomenological modelling has been previously attempted [1–4], a rigorous three-dimensional material constitutive model which is able to account for cyclic actions is lacking. Building upon previous developments available for other quasi-brittle materials such as concrete [5–7], this paper advances a consistent and detailed 3D plasticity-damage material constitutive model for wood which is able to simulate its key failure modes.

The plasticity component of the model simulates the ductile nonlinear behaviour and permanent deformation of timber under compressive stresses. Besides, modelling of brittle shear and tensile failure is based on Continuum Damage Mechanics (CDM) theory which, although not suitable for explicit crack representation, allows the monitoring of damage evolution and the identification of potential rupture zones through a smeared continuous approach. The coupling of plasticity and damage models is particularly attractive for materials with an inelastic behaviour characterized by the simultaneous occurrence of plastic flow and cracks formation. Moreover, plasticity-damage models are able to reproduce the material stiffness degradation characteristic of cyclic loading problems subjected to extensive stress redistribution more accurately [7,8]. Nevertheless, in order to obtain reliable results, the thermodynamic consistency of the model should be verified to ensure that energy is dissipated and to avoid the introduction of spurious energy into the system [6].

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In recent years, a few attempts have been made to develop a material constitutive model for timber subjected to monotonic loading that can deal with both ductile and brittle failure modes. However, to the authors' knowledge, no research has been carried out on timber constitutive models tailored to reproduce the cyclic response of wood. And while most of the available timber models for monotonic loading employ plasticity theory for failure in compression, two different approaches have been typically followed for shear and tension related failure modes. The first approach [9,10] is based on nonlinear fracture theory for the development of cohesive zone models, and aims to simulate explicitly the formation and growth of cracks. The major disadvantages of this method are the complexity of the formulation and the practical difficulties associated with the determination of appropriate values for the input parameters. Furthermore, its implementation is generally feasible only for discrete crack modelling, which requires an *a priori* definition of the cracking path, severely limiting its applicability to simulate most timber joints (commonly dowelled connections) subjected to cyclic loading where the crack locations are not known in advance. The second approach, and the one followed herein, is based on CDM theory [11–13] and represents a practical alternative for the definition of a plasticity-damage model for the analysis of timber structures subjected to load reversals.

Among the various existing plasticity models for anisotropic materials, Hill [14] and Hoffman [15] have been frequently employed for modelling timber failure under monotonic loads. For example, Kharouf et al. [16] employed Hill's criterion to develop a 2D elasto-plastic orthotropic model with anisotropic hardening, capable of simulating the biaxial (perpendicular and parallel to the grain) behaviour of timber under compression stresses only. Plastic softening and hardening was defined in the directions parallel and perpendicular to the grain, respectively. Brittle failure modes (tension or shear) were not considered by the authors and their model was employed in the analysis of the monotonic response of timber bolted connections. Later, Xu et al. [12] developed a timber model based on a combination of anisotropic plasticity with hardening for compression, and a simplified continuous damage model for shear and tension. Hill's yield criterion was used for plasticity, whereas a modified version of the same criterion, considering tension and shear stresses only, was used for the definition of the onset of damage. A simplification of the damage evolution law was introduced through a direct reduction of the elastic modulus in the three material orthogonal directions. This plasticity-damage model was used to study the behaviour of timber-steel dowelled joints subjected to monotonic tension loads only. Previously, the same model had been used to study the embedding strength of Glulam dowelled connections [17]. The model prediction accuracy achieved in both studies was satisfactory. However, its lack of continuous damage evolution laws and loading-unloading conditions render this model unsuitable for load reversal simulation.

On the other hand, Sandhaas et al. [13] implemented a constitutive model for wood under monotonic loads based exclusively on CDM. The accuracy of this damage model was evaluated against experimental results of timber specimens subjected to monotonic tension, compression and dowel embedment. The model takes into account eight types of brittle and ductile failure modes, each of them associated with a different failure criterion. In spite of not including plasticity, the explicit definition of linear softening (tension) and perfectly plastic (compression) damage evolution laws, as a function of an internal threshold variable controlling the size of the damage surface, constitutes an important contribution for the numerical modelling of wood. More recently, in order to study the brittle failure modes of dowelled timber-steel connections subjected to monotonic tension loading, a new timber model was developed by Khelifa et al. [11] within the framework of plasticity coupled with CDM. This refined constitutive model incorporates the effects of orthotropic elasticity, anisotropic plasticity with isotropic hardening, isotropic ductile damage, and large plastic deformations. Nonetheless, the model does not consider different input strength parameters for tension and compression failure. Conversely, only one set of average strength parameters are required to be calibrated for each particular problem. Moreover, only one Hill's surface is used simultaneously as plasticity and damage criteria, making both inelastic behaviours totally dependent on one another. All these characteristics make this model also unsuitable for the study of the nonlinear response of timber subjected to cyclic loading.

This paper describes the implementation of a 3D material constitutive model for wood that is capable of reproducing its cyclic response and failure modes, through the coupling of a general orthotropic plasticity model with isotropic hardening, and an isotropic continuous damage model. To this end, the next section discusses the nonlinear experimental response and failure characteristics of wood under compression, tension and shear stresses. This is followed by a presentation of the theoretical basis and constitutive equations of the elasticity and damage parts of the proposed model. The general orthotropic plasticity formulation is developed and the computational aspects of the model implementation are discussed, including the coupled plasticity-damage algorithm and the derivation of the algorithmic consistent tangent stiffness matrix. Subsequently, the thermodynamic consistency of the model is verified and the ability of the model to simulate the uniaxial cyclic response of timber in the directions parallel and perpendicular to the grain is demonstrated. Finally, the experimental response of a timber-steel dowelled connection subjected to cyclic loading is employed in order to validate the proposed model, and general conclusions are outlined.

2. Background on the nonlinear behaviour of wood

Wood is an anisotropic material with different types of failure modes in shear, tension and compression depending on the loading direction relative to its grain alignment. This means that the mechanical properties of wood (e.g. elastic moduli, shear moduli, Poisson's ratio and strength) vary with the directions and signs (i.e. whether in tension or compression). In general, the three most important failure modes are [12]: (i) ductile failure due to compression parallel to the grain, (ii) ductile failure due to compression perpendicular to the grain, and (iii) brittle failure due to shear parallel to the grain and tension perpendicular to the grain.

Fig. 1 shows the compressive and tensile stress-strain curves of Scandinavian spruce with a mean density of 430 kg/m^3 obtained experimentally by Karagiannis et al. [3]. It can be appreciated from this figure that, for the direction parallel to the grain (Fig. 1a), the behaviour in compression is approximately linear elastic until the compressive strength is reached at around 40 MPa. After this point, a minor stress drop is produced followed by a plastic plateau. Alternatively, the compressive stress-strain relationship in the direction perpendicular to the grain (Fig. 1b) shows plastic behaviour with moderate hardening. It is important to note that the compressive strength in the direction perpendicular to the grain is less than 10% of the strength in the direction parallel to the grain. In the case of tension (Fig. 1c and d), an initial linear elastic response is followed by a brittle failure in both directions. Therefore, the post-elastic behaviour of wood in tension is markedly different than that in compression due to absence of plastic deformation and the sudden loss of strength at failure. Finally, even though the shear stress–strain curve in the direction parallel to the grain is nonlinear, a brittle failure is also observed in this case (Fig. 2). In light of the above discussion, timber failure modes can be summarized as ductile elastic–plastic failure with large deformation for compressive stresses, and elastic brittle failure for the interac-