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Topology optimization of thin plate structures with bending stress constraints

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ABSTRACT

This paper presents the topology optimization of thin plate structures with bending stress constraints. To avoid the stress singularity phenomena, the qp-relaxation is used for local stress interpolation. The local stress constraints are aggregated into a single global constraint based on the p-norm stress measure. The framework of the topology optimization is constructed using the commercial finite element software ANSYS. In the presented work, the volume of the structure is minimized with the global stress constraint. Numerical examples are demonstrated to validate the proposed topology optimization method.

1. Introduction

Topology optimization, which is one of the most popular optimization methods, is widely used for the conceptual design of structures. A majority of studies on topology optimization have been conducted to maximize the stiffness of the structures with a limited amount of materials [1–3]. However, topology optimization for maximum stiffness design may not satisfy the material failure criterion, such as a maximum allowable stress. For this reason, the necessity of employing a topology optimization method that considers stress constraints has emerged [4,5].

Many studies that regard stress-constrained topology optimization have been conducted over the past two decades. Among them, active research has been conducted to resolve the stress singularity problem and handle numerous local stress constraints. The stress singularity problem is originally encountered in the optimization of truss structures that have stress constraints [5–7]. The ε-relaxation approach [8], which relaxes the stress constraints, was proposed to solve the stress singularity problem, and stress-constrained topology optimization based on the SIMP (Solid Isotropic Material with Penalization) method was developed using local stress interpolation and ε-relaxation [9]. The qp-relaxation [10] was also proposed to solve the stress singularity problem in SIMP-based topology optimization with stress constraints. The local stress constraints require a large amount of computational effort and exhibit highly nonlinear characteristics. To address these difficulties, constraint aggregation methods using p-norm [11–13] or the Kreisselmeier-Steinhauser (KS) function [14–17] were developed to handle local stress constraints effectively. Recent studies also present the topology optimization for the reinforcement design considering stress constraints [18–20].

The above-mentioned studies on stress-constrained topology optimization have been conducted using the structures that were discretized using plane or solid element types. However, in real engineering problems, a structure is often discretized using plate elements based on the behavior of deformation. In this respect, topology optimization of plate structures for compliance minimization, eigenvalue maximization and reinforcement design have also been studied for a long time [21–27]. The design of reinforcement in plate structures including stress constraint also has been reported [28]. A recent study proposed the thickness optimization of the plate structure under stress constraints to consider failure of the structures [29]. Nevertheless, topology optimization of the plate structures with bending stress constraints has not been reported to date in the literature.

In this research, we propose the topology optimization of thin plate structures while considering bending stress constraints for the first time. The plate structure is discretized using the discrete Kirchhoff triangular (DKT) element [30]. The qp-relaxation is applied in the process of the element stress calculation to resolve the stress singularity problem. The global stress constraint using the stress p-norm is used to handle the local stress constraints effectively. Sensitivity analysis of the global stress constraint is performed by the adjoint variable method. In this paper, the implementation of the topology optimization was conducted by the commercial finite element software ANSYS. Several numerical

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examples are presented to validate the proposed topology optimization method.

2. Finite element discretization of thin plates

In this research, the DKT element is used for the discretization of thin plate structures. This element is a 3-node, 9 degrees-of-freedom plate element, as shown in Fig. 1.

The displacement components of the plate can be assumed by Eq. (1) to be based on plate theory with a transverse shear strain [31,32]

$$u = z\beta_x(x,y), \quad v = z\beta_y(x,y), \quad w = w(x,y) \tag{1}$$

where u and v are the displacement in the x and y directions, respectively, w is the transverse displacement, z is the distance from the middle surface ($-h/2 \leqslant z \leqslant h/2$), and h is the thickness of the plate. The variables β_x and β_y are the rotations of the normal to the undeformed middle surface in the $x-z$ and $y-z$ planes, respectively. With the displacement assumptions in Eq. (1), the bending strain $\boldsymbol{\varepsilon}_b$ can be given by

$$\boldsymbol{\varepsilon}_b = z\boldsymbol{\kappa}, \quad \boldsymbol{\kappa} = \begin{bmatrix} \frac{\partial \beta_x}{\partial x} \\ \frac{\partial \beta_y}{\partial y} \\ \frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x} \end{bmatrix} \tag{2}$$

where $\boldsymbol{\kappa}$ is the vector of curvature components. In the DKT element, the Kirchhoff hypothesis is imposed on discrete points to model the thin plate. In this instance, the discrete points are each node and mid-node of the element [30]. Consequently, the curvature vector $\boldsymbol{\kappa}$ can be expressed by the following matrix equation

$$\boldsymbol{\kappa} = \mathbf{Bu} \tag{3}$$

where $\mathbf{B}$ is the strain-displacement matrix, and $\mathbf{u}$ is the vector of nodal displacement. Therefore, the stiffness matrix of the DKT element can be expressed as follows based on the bending strain energy of the plate [33]:

$$\mathbf{K}_{DKT} = \int_A \mathbf{B}^{\mathrm{T}}\mathbf{D}_0\mathbf{B}\,dA \tag{4}$$

$$\mathbf{D}_0 = \frac{h^3}{12}\cdot \mathbf{C}_0 \tag{5}$$

$$\mathbf{C}_0 = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{6}$$

where A, E and ν are the area of the element, Young's modulus and Poisson's ratio, respectively. The notation $\mathbf{C}_0$ is the constitutive matrix for the plane stress. The details for the derivation of the stiffness matrix of the DKT element are described in [30].

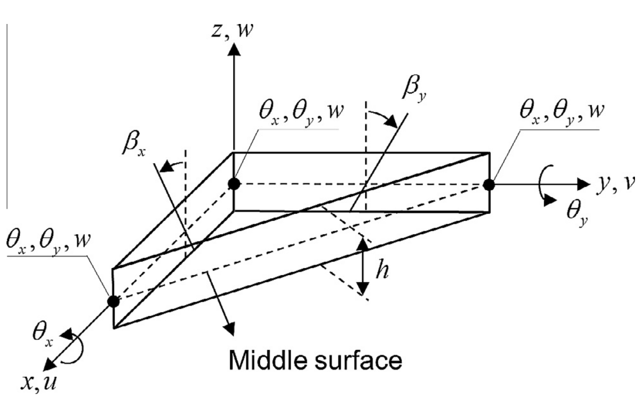

Fig. 1. Deformation assumptions for a plate element.

3. Formulation of topology optimization

3.1. Statement of the optimization problem

The SIMP-based topology optimization problem that minimizes the volume subject to local stress constraints can be described as follows:

$$\begin{aligned} \min_{\rho} \quad & V = \sum_{e=1}^{NE} \rho_e v_e \\ s.t. \quad & \sigma_e^{VM} \leqslant \bar{\sigma} \quad (e = 1, 2, \ldots, NE) \\ & \mathbf{Ku} = \mathbf{f} \\ & 0 < \rho_{\min} \leqslant \rho_e \leqslant 1 \end{aligned} \tag{7}$$

where ρ_e, σ_e^{VM} and v_e are the design variables, von-Mises stress and volume of the e-th element, respectively. The variable $\bar{\sigma}$ is a maximum allowable stress, and NE is the number of elements. In the SIMP method, the constitutive matrix of the e-th element $\mathbf{C}_e$ can be defined as

$$\mathbf{C}_e = \rho_e^p \mathbf{C}_0 \tag{8}$$

where p is the penalization factor. The design element is solid material when the design variable is 1 and void when the design variable is 0. For the intermediate value of the design variable, the constitutive matrix of the design element is interpolated using Eq. (8). To avoid the singular matrix problem in the finite element analysis, a lower bound for the design variable takes on a notably small but finite value, such as $\rho_{\min} = 10^{-3}$.

The local stress constraints in Eq. (7) have the stress singularity problem, which arises from the discontinuity of the stress constraints [8]. In this research, the qp-relaxation [10] is applied to solve the stress singularity problem. Note that the degeneracy of the stress constraint is not affected by the evaluation of bending stress at the top or bottom surface. Hence, the singularity of stress constraints in plate bending problem can be resolved by the qp-relaxation as in the plane stress problem. In the thin plate, the bending stress of the e-th element $\boldsymbol{\sigma}_e$ at the top surface ($z = h/2$) can be represented as follows using qp-relaxation:

$$\boldsymbol{\sigma}_e = \frac{\mathbf{C}_e \boldsymbol{\varepsilon}_b|_{z=h/2}}{\rho_e^q} = \rho_e^{p-q}\mathbf{C}_0\boldsymbol{\varepsilon}_b|_{z=h/2} = \frac{h}{2}\rho_e^{p-q}\mathbf{C}_0\mathbf{B}_e\mathbf{u}_e \tag{9}$$

where $\mathbf{B}_e$ and $\mathbf{u}_e$ are the strain-displacement matrix and the nodal displacement vector of the e-th element, respectively. In this paper, the values of $p = 3$ and $q = 2.5$ are used for the qp-relaxation.

Because the optimization problem Eq. (7) considers the stress constraints of each design element, the number of constraints is equal to the number of design elements. Thus, even with the adjoint variable method, a large amount of computational effort is required to calculate the sensitivities of the local stress constraints. Furthermore, local stress constraints exhibit highly nonlinear characteristics. For these reasons, the global stress constraint using the stress p-norm [12,34] is used instead of local stress constraints. Thus, the optimization problem in Eq. (7) becomes

$$\begin{aligned} \min_{\rho} \quad & V = \sum_{e=1}^{NE} \tilde{\rho}_e v_e \\ s.t. \quad & c^I \sigma_{PN} \leqslant 1 \\ & \mathbf{Ku} = \mathbf{f} \\ & 0 < \rho_{\min} \leqslant \rho_e \leqslant 1 \end{aligned} \tag{10}$$

where