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Limit Analysis of a historical iron arch bridge. Formulation and computational implementation

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ABSTRACT

In the paper an innovative formulation of a FEM computational model for the analysis of elastoplastic 3D truss-frame structures is developed within the framework of the Theory of Plasticity and Limit Analysis. Reference application is made to a historical iron arch bridge. Original computational features implemented in the solving algorithm allow for tracking the limit structural behavior of the bridge, by reaching convergence with smooth runs up to the true limit load and by tracing the corresponding collapse displacements. This is achieved disregarding for the considerable number of dofs of the whole structural model, ranging in the order of thirteen thousands.

1. Introduction

In this paper, an autonomously-developed elastoplastic formulation and implementation in a computer code apt to model general 3D truss-frame structures has been outlined and applied to develop a complete non-linear structural analysis of a marvelous historical iron arch bridge, namely the Paderno d'Adda bridge, built in 1889 over the river Adda near Milano, northern Italy. Specific accounts on the various features of this remarkable structure are available in SNOS [1], Nascè et al. [2], Nascè [3], and further gathered in Ferrari [4], Ferrari et al. [5–8] (brief information is presented here in Section 4). The code runs within a MATLAB environment and allows for tracking the true limit load multiplier, by a sophisticated computational strategy apt to describe the "exact" evolutive response of the structure until collapse, according to classical concepts from the Theory of Plasticity and Limit Analysis, as applied to the inelastic analysis of beam and frame structural systems (see e.g. Massonet and Save [9], Kaliszky [10], Jiràsek and Bažant [11]).

Elastoplastic analysis of frames has been the subject of considerable research interests since the sixties (see e.g. Maier [12,13], Maier et al. [14], Capurso [15], Hodge [16], Tin-Loi [17], Zouain et al. [18], Kaliszky and Lógó [19], Olsen [20], Liew et al. [21], Kaliszky and Lógó [22]), also in view of developing robust and efficient computational algorithms (see e.g. Maier [23], Papadrakakis and Papadopoulos [24], Van Long and Nguyen-Dang [25]), including in the framework of so-called "direct methods" (see e.g. contributions in Spiliopoulos and Weichert [26] and Forward by G. Maier on it), which are not of a primary concern in the present paper.

Further developments along this line of research during the last decade are many. Among them, a brief presentation on representative examples follows. Tangaramvong and Tin-Loi [27] treated the limit load estimation under the effect of combined stresses, considering local strain softening and both path-independent (holonomic) and path-dependent (nonholonomic) behaviors. Skordeli and Bisbos [28] and Manola and Koumousis [29] proposed methods for approximating the yield surface with ellipsoids and with a polyhedron expressed in the context of a convex hull, respectively, instead of considering the delimitation of the elastic domain as piecewise linearized. Mahini et al. [30] proposed a method to determine the plastic multipliers of perfectly elastoplastic frames as a solution of a Linear Programming (LP) problem by exploiting the plastic work criteria, referred to as Dissipated Energy Minimization approach. Later, such method has been extended by the same authors to softening frames (Mahini et al. [31]). In Thai and Kim [32] and Dehghani et al. [33] the possibility to consider the plastic deformation varying through the cross section and along the length of the structural members have been explored. Yang et al. [34] presented a work in which the effects of uncertainties are considered, as related to the applied forces and to the plastic material capacities of the structure.

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Resting on the unifying basis provided by the Linear Complementarity Problem (LCP) (see Cottle et al. [35], Billups and Murty [36]), a specific account of which is available, together with its frequent involvement in the theory of elastoplasticity, e.g. in Maier [23], Cohn and Maier [37], Franchi and Cohn [38], Lloyd Smith [39], Wakefield and Tin-Loi [40], Giambanco [41], the procedure involved in the present algorithm starts from earlier contributions gathered in Cocchetti and Maier [42], therein focused on the analysis of softening frames, globally referred to here as "exact time integration" method.

In the work presented herein, such a procedure is further developed in theoretical terms, to be finally applied, for the first time, to the elastoplastic analysis of a perfectly-plastic large truss-frame structure with considerable complexity, namely a 3D FEM model of the Paderno d'Adda bridge. This involves roughly 5,300 beam finite elements with plastic joints and 13,300 degrees of freedom.

In light of the complexity and size of the structures to which the presented algorithm is addressed, some simplifying assumptions have been adopted towards practical and efficient implementation and use. These refer to: lumping plasticity at some pre-selected sections ("plastic joints", as "plastic hinges") and using Piece-Wise-Linear (PWL) elastic perfectly–plastic models to describe the behavior of such critical sections. Moreover, the non-linear problem is numerically solved on the basis of the "step-wise holonomic" interpretation of the evolution of a dissipative system (Maier [23], De Donato and Maier [43], Franchi and Genna [44]). This means that the dependence on the given loading path does not hold within the current step; in other terms, the intrinsic irreversibility of the plastic hinge model is accounted for by updating only the internal variables at the transition stage from step to step.

Furthermore, the formulation presents several peculiar features that appear to be innovative, both in general terms and in their specific application to the present structural context. They refer to: (i) a new approach for solving structural systems for truss-frame analysis modeled by finite elements, with particular attention to the imposition of kinematic constraints; (ii) the determination of the global elastoplastic matrix of the structure that is involved in the solving system, based on an iterative "updating" procedure; and (iii) the application of such a procedure to the effective modeling of the global non-linear elastoplastic behavior of a large truss-frame structure, like that of the Paderno d'Adda bridge.

The paper is structured as follows. Section 2 presents a new approach for solving structural systems for truss-frame analysis modeled by finite elements, within the elastoplastic range. In particular, the formulation adopted in the solving procedure, based on a tangent stiffness formulation, is described. Section 3 outlines specifications about the determination of the tangent stiffness matrix, when the yield domain is represented by uncoupled conditions. In such a case, the tangent stiffness matrix is determined through a dedicated Gaussian elimination procedure. Section 3 presents as well the sub-step incrementation implemented in the driving algorithm. The main steps of the incremental procedure are pointed out and then schematically resumed in a flow-chart. Section 4 illustrates the main results of first complete elastoplastic analyses of the Paderno d'Adda bridge. Brief comments on various computational aspects and effectiveness of the simulations are concisely pointed out in closing Section 5.

Matrix notation is adopted throughout. Matrices and vectors are represented by bold-face symbols. Transposition is indicated by superscript *T*. A dot marks a time rate, i.e. a derivative with respect to an ordering, not necessarily physical, time variable *t*.

2. Formulation and general framework of the computational algorithm

In following Section 2.1 cornerstone equations from the Theory of Plasticity and Limit Analysis, as strictly pertinent to the present context, are arranged, to present a new approach of solving system for truss-frame analysis, further described in Section 2.2. Although the present implementation has been fully developed in a 3D setting, simpler 2D space representations will be adopted for illustration purposes.

2.1. Formulation in the framework of the Theory of Plasticity and Limit Analysis

According to traditional FEM modeling in structural engineering, truss-frame structures are modeled by conventional finite elements, i.e. the structure is assumed to be the assembly of various elements connected at a discrete number of points (nodes). External forces and constraints are reduced to act at these points. In particular, the present FEM formulation is based on classical Euler–Bernoulli beam finite elements, according to the following peculiar hypotheses: straight elements, uniform cross sections, homogeneous material properties, transverse displacements modeled by cubic shape functions (i.e. negligible shear strain effects), axial rotations and displacements varying linearly along the beam element.

Moreover, possible inelastic deformations manifest themselves only at pre-selected sections (plastic joints). As a generalization of the classical plastic hinge concept in the Limit Analysis of frames (see e.g. Cocchetti and Maier [42] and references quoted therein) it is assumed that critical sections are located between adjacent conventional linear elastic finite elements. The behavior of the critical sections is described by an elastoplastic (perfectly-plastic) model with PWL yield functions. This means that the relationships between static and kinematic generalized variables of the beam cross sections are piecewise linearized (see e.g. Maier [13,23], Capurso [15], Hodge [16], Tin-Loi [17], Olsen [20]).

According to the idealizations above, the constitutive relations below describe a general formulation of a PWL elastoplastic model in the critical section of a beam element (all relations are stated in terms of total quantities):

$$\mathbf{q} = \mathbf{e} + \boldsymbol{\eta}, \quad \mathbf{e} = \mathbf{k}^{-1}\mathbf{N}, \quad \boldsymbol{\eta} = \mathbf{n}\boldsymbol{\lambda} \tag{1}$$

$$\boldsymbol{\varphi} = \mathbf{n}^T\mathbf{N} - \mathbf{Y} \leqslant \mathbf{0}, \quad \boldsymbol{\lambda} \geqslant \mathbf{0}, \quad \boldsymbol{\varphi}^T\boldsymbol{\lambda} = 0 \tag{2}$$

Eqs. (1) and (2) relate the generalized strain history $\mathbf{q}$ to the generalized stress history $\mathbf{N}$. With reference to a 2D interpretation of truss-frames (Fig. 1), the generalized strains in vector $\mathbf{q}$ consist of relative rotations at the extremities of the beam (with respect to a principal centroidal axis of the cross section) and axial elongation (discontinuity of displacement along the centroidal axis of the

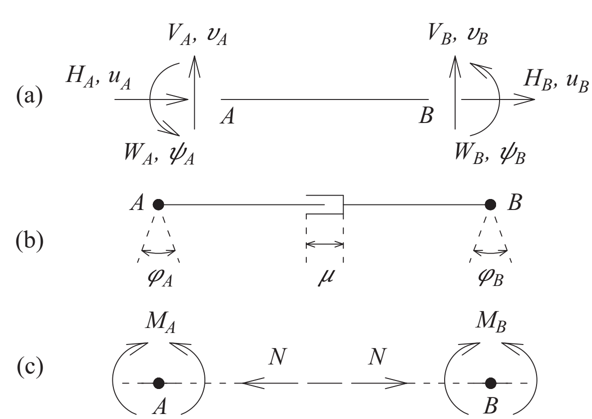

Fig. 1. Schematic description of a (2D) beam finite element with plastic joints: (a) Static (external) variables $\mathbf{H}$ and kinematic (external) variables $\mathbf{u}$. (b) Kinematic plastic internal variables $\boldsymbol{\eta}$ at the plastic joints. (c) Static internal variables $\mathbf{N}$.