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A plastic-damage-contact constitutive model for concrete with smoothed evolution functions

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ABSTRACT

A new 3D finite element concrete model is described. The model brings together two recently developed sub-models for simulating cracking and crack contact behaviour, both of which use smoothed evolution functions, with a triaxial plasticity model component. A number of examples are presented that validate the model using a range of plain and reinforced concrete test data. These examples demonstrate that the model is numerically robust, has good equilibrium convergence performance and is objective with respect to mesh grading and increment size. The examples also illustrate the model's ability to predict peak loads, failure modes and post-peak responses.

1. Introduction

The complex nature of the mechanical behaviour of concrete has provided numerical researchers with a set of problems that nearly fifty years of work has yet to fully resolve. The multi-scale nature of this particulate material and the many mechanical mechanisms that govern its behaviour combine to make the development of a comprehensive finite element concrete model a truly challenging undertaking. Inherent flaws in early concrete models [1] became apparent in the mid 1980s as the fundamental importance of scale effects and the need for fracture mechanics concepts came to be understood [2,3]. From the finite element view point, this work demonstrated that it was not possible to use a constitutive model for softening behaviour governed by a unique stress–strain function whilst also maintaining objectivity with respect to mesh grading. Thus, it was recognised that the constitutive and computational aspects of a solution have to be considered together.

We use the word *constitutive* to describe the stress–strain behaviour of a representative volume of material with the characteristic dimension of the fracture process zone (FPZ) width. The term *computational* encompasses numerical aspects of behaviour associated with the spatial and temporal discretisations. Thus, if the predicted behaviour changes with the density of a finite element mesh, we would describe this as a computational issue. It is recognised that the underlying micro- and nano-structure of a particular material has been (and should be) used in the development of computational techniques, which implies that there should not be such a clear distinction between what we describe as *constitutive* and *computational* aspects of modelling. Whilst this is true, we have – for convenience – continued to use these terms in our descriptions of previous numerical models.

Bazant and Oh's Crack Band model [3] was the first to address the issue of mesh dependency. Since the publication of their work, there has been a number of important computational developments that have combined constitutive and computational aspects of the modelling of concrete. These have included integral and gradient non-local models [4–12], visco regularisation schemes [13,14], the extended finite element approach (X-FEM) [15–17], multi-scale models [18,19] elements with embedded strong discontinuities [20–25] and elements with other enhanced interpolations to overcome problems with mesh bias [26]. Most frequently, these methods have been implemented with damage models [27,28], plasticity models [29,30] or plastic-damage models [31]. These references represent only a small fraction of the research undertaken on modelling of concrete structures but in spite of the considerable progress made on this topic, not all of the computational problems have been fully resolved.

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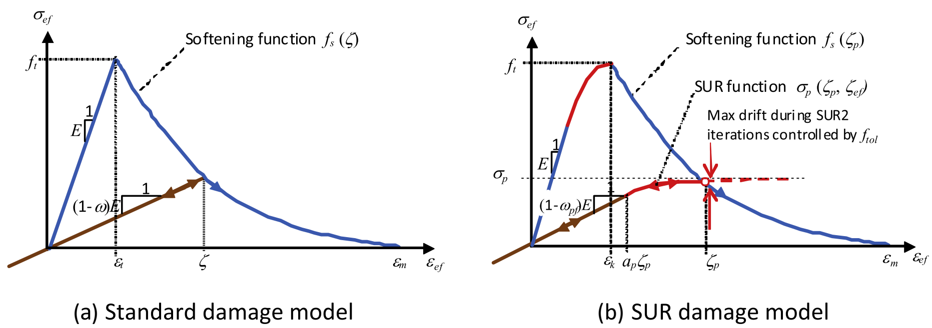

Fig. 1. (a) Standard damage model. (b) SUR damage model.

Whatever the combined constitutive-computational procedure adopted, the resulting non-linear systems of equations are most-frequently solved using Newton-based incremental–iterative schemes [32,33]. The authors' experience with many of the above computational techniques (i.e. viscous regularisation, integral and gradient non-local schemes and procedures which model strong discontinuities) is that situations can arise in which there is breakdown of the incremental–iterative scheme, such that equilibrium convergence cannot be achieved to an acceptable tolerance. This is true for standard Newton and modified Newton methods, solutions with automatic step selection, line-search solutions as well as solutions with various types of arc-length control [32,33].

In an attempt to avoid such difficulties, a number of researchers have developed solution procedures that either avoid (or limit) the use of iterations. These methods include the 'implicit–explicit' approach of Oliver et al. [34,35] in which a projected state variable (e.g. a damage parameter) is used to determine a predicted consistent tangent matrix that is exact for the current increment but for which a correction is made in the subsequent stress-recovery phase. An alternative approach, based on sequential linear solutions, was proposed by Rots et al. [36]. This method was subsequently extended by Graça-e-Costa et al. [37] such that the algorithm can capture responses from non-linear materials governed by both loading and unloading behaviour of a softening material. It has also been recently applied with a new smooth crack propagation algorithm [38].

Although there are considerable benefits to using these non-iterative approaches, they can result in non-smooth responses, and would require further development before being able to cope well with constitutive models that include non-linear crack closure in combined shear and normal modes. Currently they are not naturally compatible with non-linear plasticity models for other materials, which would be an issue for solving soil–structure problems.

Crack opening and closing behaviour is important in most concrete structures, and even when the global loading is essentially monotonic in nature, some cracks open and then close again as other cracks grow [39]. Cracks in concrete are rough in nature and contact can be regained with increasing shear displacement even when a crack has a significant opening. This shear contact, often referred to as 'aggregate interlock', is important because it can be a significant load carrying mechanism in reinforced concrete structural members [40]. Crack opening and closing under normal displacements and aggregate interlock are both aspects of crack contact behaviour that should, in the authors' opinion, be treated in a unified manner. The introduction of crack contact into a finite element concrete model can result in loss of numerical robustness because the abrupt change of stiffness that occurs upon crack closure can result in the failure of the incremental–iterative solution procedure. A fuller review of previous experimental and numerical work on modelling crack opening-closing behaviour is provided in Ref. [41].

The philosophy that underlies both of the sub-models described in this paper is that the convergence of a Newton type solution to a set of non-linear equations is generally more reliable when the equations are smooth and the tangent matrix is positive-definite than when the equations are discontinuous and/or the tangent matrix is non-positive-definite. The processes involved with concrete cracking and crack closure may appear to be naturally discontinuous and not readily amenable to smoothing; however, a close examination of the experimental response of concrete elements [42] suggests that these processes are not truly abrupt. This means that there is scope for using smoothed crack evolution and crack contact functions. This thinking led to two separate developments. The first of these was a new model that employs smooth crack contact and contact evolution functions to simulate rough crack contact behaviour in concrete [41]. The authors showed how this approach could be applied in an anisotropic damage model using embedded crack planes. The computational benefits of smoothing crack closure paths has also been demonstrated by Sellier et al. [43]. The second development [44] was a new approach to the simulation of damage evolution using a smoothed unloading–reloading response function within an algorithm that always uses a positive definite stiffness matrix. This approach was applied with an isotropic damage model and was shown to give considerable benefits in terms of the efficiency of the incremental iterative solution process.

The work described in the present paper brings together the two developments described in Refs. [41,44], along with a plasticity model component [45,46], to form a new comprehensive 3D plastic-damage-contact model for concrete. There was considerable new work involved in making the two approaches compatible with one another and with the triaxial plasticity component, as well as with the development of a robust consistent solution algorithm for the new combined smoothed plastic-damage-contact model.

Many previous papers have described the analysis of plain concrete fracture specimens, but it is rare to find descriptions of the analysis of reinforced concrete (RC) members that include mesh convergence studies, consideration of both pre- and post-peak responses and an examination of failure modes. Although models have long since been able to obtain a reasonable pre-peak response (with some model calibration), the ability of models to simulate all of the preceding facets of behaviour is unclear. The authors' experience is that the multiple local instabilities that occur in the analysis of RC members commonly make it difficult to obtain reliable, comprehensive and accurate simulations for RC members, particularly if dominant shear cracks occur. The examples presented in this paper address all of these issues and provide a thorough examination of the performance of the model with respect to the analysis of both plain and reinforced concrete structural elements.