Computers and Structures 169 (2016) 69–80

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Application of the three-node triangular element with continuous nodal stress for free vibration analysis

Yongtao Yang [a,*], Dongdong Xu [b], Hong Zheng [a]

[a] State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan, China
[b] Key Laboratory of Geotechnical Mechanics and Engineering of Ministry of Water Resources, Yangtze River Scientific Research Institute, Wuhan, China

ARTICLE INFO

Article history:
Received 28 October 2014
Accepted 19 March 2016

Keywords:
Partition of unity method
Hybrid FE-Mesh free element
Trig3-CNS
Mesh distortion

ABSTRACT

A three-node triangular element with continuous nodal stress (Trig3-CNS) was recently proposed for static analysis. The Trig3-CNS element, which is the development of the partition-of-unity based "FE-Meshfree" quadrilateral element with continuous nodal stress (Quad4-CNS), uses hybrid shape functions that combine the meshfree and finite element shape functions so as to synergize the individual strengths of meshfree and finite element methods. As a result, high order global approximations in Trig3-CNS element could be easily constructed without adding extra nodes and DOFs, thereby achieving high accuracy and convergence rate. In this paper, the element is further applied to conduct free vibration analysis of two-dimensional solids. The numerical tests in the present work demonstrate that Trig3-CNS has higher tolerance to mesh distortion and gives more accurate solution as compared to the three-node triangular element (Trig3) and four-node quadrilateral element (Quad4).

1. Introduction

The FEM [1], a well-established and powerful numerical procedure, has been successfully used in various fields, including mining engineering [2–4] and surgery simulation [5]. However, the gradient of the FEM with standard DOFs often exhibits discontinuity at element boundaries and nodes. Extra smoothing operations are required to calculate nodal stress in post processing. Moreover, deterioration of accuracy of solution due to meth distortion [6] hinders the application of some types of iso-parametric elements to large deformation analysis [7].

Meshfree methods, which do not need a mesh to discretize the problem domain, provide an effective approach to solve the problems related to large deformation [8,9], impact-induced failure [10] and fracture propagation simulation [11,12]. It has been shown by Bordas et al. [13] that meshfree methods can be exploited to model arbitrary three-dimensional crack initiation, propagation, branching and junction in non-linear materials. A review of meshfree methods and implementation aspects can be found in [14]. The first meshfree method named as Smoothed Particle Hydrodynamics (SPH) was proposed in 1970s [15,16] to simulate astronomic phenomena. Then, Diffuse Element Method (DEM) [17] and Element-Free Galerkin method (EFG) [18–20] were developed on the basis of moving least square approximations and Galerkin method. Further development of meshfree method can be found in the work associated to point interpolation method (PIM) [21], reproducing kernel particle method (RKPM) [22], stable particle methods [23] and meshfree local Petrov–Galerkin method (MLPG) [24]. Meshfree methods are also not free from drawbacks. In some of the meshfree methods, extra operations are required to treat the essential boundary conditions due to the absence of the Kronecker delta property of shape functions. Besides, meshfree methods are computationally more expensive than FEM. To impose essential boundary conditions, many methods have been proposed for meshfree methods, such as Lagrange multiplier method [25], penalty method [26], modified variational principles [18] and perturbed Lagrange [27].

A family of smoothed finite element methods (S-FEMs) [28–30] including cell-based S-FEM (CS-FEM) [31], node-based S-FEM (NS-FEM) [32], edge-based S-FEM (ES-FEM) [28,33,34], and face-based S-FEM (FS-FEM) [35], which relied on strain smoothing technique [36], has been developed in recent years. Since the advent, the S-FEMs [31,37] have been explored in many complex problems: fracture mechanics [33], plate [38–41] and shell structures, elastostatics, elastoplasticity and viscoplasticity [42,31]. In order to expand the application range, S-FEMs can also be coupled to partition of unity enrichment [43,33]. Since S-FEM is able to provide an upper bound solution for solid mechanics problems [30], it is complementary to the FEM. In other front to improve FEM, polygonal

* Corresponding author.
E-mail address: scuhhc@126.com (Y. Yang).

http://dx.doi.org/10.1016/j.compstruc.2016.03.008
0045-7949/

Finite Element Method (PFEM) [44,45] has been developed. PFEM is able to construct proper approximations on polygonal elements, and provides an effective approach to remesh and refinement in two dimensions [46]. PFEM has been successfully used to conduct crack propagation analysis [46] and nonlinear thermal analysis with application to hybrid laser welding [47].

During the last two decades, several methods including the hp clouds [48], generalized finite element method (GFEM) [49], particle-partition of unity method [50], numerical manifold method [51–54] and extended finite element method (XFEM) [55], which are based on the concept of partition of unity method (PUM) [56,57], have been developed and successfully applied in many fields, including solid mechanic [58], fluid mechanic [59] and heat transfer [60,61]. PUM provides a convenient approach to construct high order global approximations without adding extra nodes, thereby achieving high accuracy and convergence rate [62]. However, PUM has a serious disadvantage, known in the literatures as "linear dependence" (LD) problem [63]. It occurs when both the PU functions and the local approximation functions are taken as explicit polynomials. Some effective approaches to eliminate the LD problem can be found in [64].

To synergize the individual strengths of meshfree and finite element methods, a new family of elements called 'FE-Meshfree' elements which is based on the PUM has been developed [7,62,63,65–68], such as the hybrid FE-Meshfree four-node quadrilateral element (FE-LSPIM QUAD4) [65], FE-Meshfree three-node triangular element (FE-LSPIM Trig3) [63]. The hybrid shape functions of these "FE-Meshfree" elements are constructed by combining the meshfree and finite element shape functions. As a result, these "FE-Meshfree" elements compared to FEM inherit better accuracy, higher convergence rate, and higher tolerance to mesh distortion from the meshfree methods, while preserving the Kronecker-delta property of FEM. Moreover, these "FE-Meshfree" elements have been known to be free from the linear dependence problem which otherwise cripples many of the PU based finite elements [63]. However, like the FEM with standard DOFs, the gradient of the FE-Meshfree elements proposed by Rajendran et al. [7,62,63,65–68] is not continuous at the nodes, and extra smoothing operations are required to calculate nodal stress in post processing. To improve the property of FE-LSPIM QUAD4, Tang et al. [69] developed a new hybrid FE-Meshfree four-node quadrilateral element with continuous nodal stress (Quad4-CNS). Furthermore, a hybrid FE-Meshfree three-node triangular element with continuous nodal stress (Trig3-CNS) [70] was developed. This triangular element, Trig3-CNS, is potential to replace Quad4-CNS [69] and the hybrid FE-Meshfree four-node quadrilateral element (FE-LSPIM QUAD4) [65] when the geometry of computational domain is too complex to generate quadrilateral meshes. Trig3-CNS also can be regarded as the development of the PU-based FE-Meshfree three-node triangular element (FE-LSPIM Trig3) [63]. Compared to FE-LSPIM Trig3, Trig3-CNS is capable of giving smoother solution and calculating nodal stress without any extra smoothing operation [70]. For the sake of differentiating Trig3-CNS from FE-LSPIM Trig3, in the rest of this paper, FE-LSPIM Trig3 will be termed as Trig3-DNS.

In other front, based on the concept of "twice-interpolation" procedure, Zheng et al. [71] proposed an improved triangular element with continuous stress for elastic problems. Application of this new element for crack propagation problems in 2D elastic solids could be found in Ref. [72]. Bui et al. [73] successfully formulated a new four-node quadrilateral element with continuous nodal stress by applying the twice-interpolation procedure. In their work, the term "twice-interpolation" is replaced by a different name as "consecutive-interpolation" for better fit. Later, they further developed an extended consecutive-interpolation quadrilateral element (XCQ4) and applied this element to linear elastic fracture mechanics [74]. It is noticed that the Trig3-CNS element developed by authors is totally different from the elements proposed by Zheng et al. [71] and Bui et al. [73]. This Trig3-CNS element is a development of the previous partition-of-unity-based "FE-Meshfree" quadrilateral element [69].

There have many effective numerical methods on the application to free vibration analysis, such as FEM [1], EFG [75], local radial point interpolation method (LRPIM) [76], the node-by-node meshless (NBNM) method [77], FE-LSPIM QUAD4 element [65], node-based S-FEM (NS-FEM) [28,31], edge-based S-FEM (ES-FEM) [28,31], extended cell-based smoothed discrete shear gap method (XCS-DSG3) [78] and iso-geometric finite element method [79–82].

Because Trig3-CNS has higher tolerance to mesh distortion and gives more accurate solution than Trig3 and Quad4 elements for linear elastic problems [70], we further explores Trig3-CNS element for 2D free vibration analysis in this study. The outline of this paper is as follows: Section 2 briefly reviews the development of shape functions for the Trig3-CNS element. Section 3 gives the mass and stiffness matrices. In Section 4, typical numerical tests are carried out to assess accuracy of the proposed Trig3-CNS element. Finally, conclusions are drawn in Section 5.

2. Construction shape function for Trig3-CNS

Consider a triangular domain Ω described by three nodes $\{P_1\ P_2\ P_3\}$ and introduce an arbitrary point $P(\boldsymbol{x})$ with the coordinates $\boldsymbol{x} = (x,\ y)$. According to the concept of PUM [56], in the triangular domain Ω, the Trig3-CNS global approximation $u^h(\boldsymbol{x})$ can be represented in the following form:

$$u^h(\boldsymbol{x}) = \sum_{i=1}^{3} w_i(\boldsymbol{x}) u_i(\boldsymbol{x}) \tag{1}$$

where $w_i(\boldsymbol{x})$ and $u_i(\boldsymbol{x})$ are the weight functions and the nodal approximations associated with node i, respectively.

Unlike Trig3-DNS element, which uses the shape functions of Trig3 to define its weight functions, the weight functions of Trig3-CNS element are written as [70]

$$w_1(\boldsymbol{x}) = L_1 + L_1^2 L_2 + L_1^2 L_3 - L_1 L_2^2 - L_1 L_3^2 \tag{2}$$

$$w_2(\boldsymbol{x}) = L_2 + L_2^2 L_3 + L_2^2 L_1 - L_2 L_3^2 - L_2 L_1^2 \tag{3}$$

$$w_3(\boldsymbol{x}) = L_3 + L_3^2 L_1 + L_3^2 L_2 - L_3 L_1^2 - L_3 L_2^2 \tag{4}$$

in which L_i are the area coordinates [1]. The transformation of the area coordinate is defined as:

$$\begin{aligned} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} &= \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix}, \begin{bmatrix} L_1 \\ L_2 \\ L_3 \end{bmatrix} \\ &= \frac{1}{2A} \begin{bmatrix} x_2 y_3 - x_3 y_2 & y_2 - y_3 & x_3 - x_2 \\ x_3 y_1 - x_1 y_3 & y_3 - y_1 & x_1 - x_3 \\ x_1 y_2 - x_2 y_1 & y_1 - y_2 & x_2 - x_1 \end{bmatrix}, \\ &\underline{\underline{\mathrm{def}}}\ \frac{1}{2A} \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} \end{aligned} \tag{5}$$

in which

$$2A = \det \begin{bmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix}, \quad L_1 + L_2 + L_3 = 1. \tag{6}$$

There are four important features for the weight functions of Trig3-CNS element as described in Appendix A.