Computers and Structures 164 (2016) 23–37

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Uncertainty propagation analysis in laminated structures with viscoelastic core

W.P. Hernández *, D.A. Castello, T.G. Ritto

Mechanical Engineering Department, COPPE, Universidade Federal do Rio de Janeiro, Rio de Janeiro, RJ, Brazil

ARTICLE INFO

Article history:
Received 23 March 2015
Accepted 22 October 2015

Keywords:
Viscoelasticity
Fractional derivative
Uncertainty propagation
Sandwich beam
Stochastic dynamics

ABSTRACT

This paper investigates how uncertainties affect the modal parameters of a sandwich structure with a viscoelastic core, which is commonly used in both passive and hybrid control strategies. The viscoelastic sandwich structural component is modelled using (1) the classical model proposed by Mead and Markus and (2) a more complex model that has recently been proposed in the literature. The mechanical behaviour of the viscoelastic core is described by fractional derivative operators. The uncertainties are assumed to come from two sources. The first source of uncertainty is associated with the physical parameters of the constitutive model used to describe the dynamic behaviour of the viscoelastic core, which should be characterised when solving the inverse problem for model calibration. The second source is associated with a set of geometrical parameters and is considered to be linked to both manufacturing processes and assembling–disassembling structural set-ups. A set of examples is performed using the Monte Carlo Simulation analysis, allowing the measurement of the impact of typical sources of uncertainties in modal predictions as well as providing means to make a comparative analysis between two viscoelastic sandwich models. Among other conclusions, it was found that the mean values of the modal parameters do not change much for the different analyses performed, and the thickness of the viscoelastic layer is the most critical variable because it affects both the modal frequency and the damping ratio.

1. Introduction

Viscoelastic sandwich structures are laminated structures which are composed by two or more rigid layers joined by soft layers that are usually made of polymeric materials. These structures have become one of the most effective ways to be used in passive vibration control strategies. The energy dissipation takes place within the viscoelastic layers mainly due to shearing strains/stresses [1]. Its dissipative characteristic has led this type of structural component to be employed in many applications in aeronautical, structural, automotive and naval industries.

The process of building a Computational Model (CM) for laminated structures demands a suitable kinematic model such that accurate strain fields can be obtained at reasonable computational costs. Many researchers have adopted the theory introduced by Mead and Markus [2] to describe the strain fields in a three-layer beam with a viscoelastic core. Concerning model predictions for damping, the model proposed by Mead and Markus presents some inaccuracies that are associated with high shear strain levels obtained within the viscoelastic core [3]. Therefore, some authors have made alternative proposals to improve the Mead and Markus theory. Douglas and Yang [4] modelled the face sheets as two independent Euler beams and included both the shear deformation and the rotary inertia for the elastic layers. The first order shear deformation theory is considered in [2–14] inasmuch as it allows both the transverse and axial displacements of the central core to be linearly dependent along its thickness. Bai and Sun [3] analysed the influence of the slip at interface bonds between layers and concluded that the constraining layers must deform independently. Bai and Sun [3] also concluded that the use of a high order theory in the modelling of the constrained viscoelastic core is needed. A review of several theories for modelling sandwich structures with applications in multilayer sandwich beams and shells are presented by Carrera and Ciuffreda [15] and by Hu et al. [16].

The more complex a CM gets, the higher the computational cost it demands. Therefore, engineers/analysts have to face a trade-off between model accuracy and computational costs when building a CM for structures containing viscoelastic laminates, viscoelastic layers or any viscoelastic subcomponents. Furthermore, no matter how complex one builds a CM, modelling uncertainties will always be one of the key-factors when using model predictions for risk analysis, decision-making processes and Engineering designs.

* Corresponding author.
E-mail addresses: wpinto@ufrj.br (W.P. Hernández), dnl.castello@gmail.com, castello@mecanica.ufrj.br (D.A. Castello), tritto@mecanica.ufrj.br (T.G. Ritto).

http://dx.doi.org/10.1016/j.compstruc.2015.10.006
0045-7949/

Uncertainties in a CM may have different sources such as manufacturing processes, natural variability, unknown boundary conditions, and unknown excitation forces. Some of these can be mitigated by deeper investigations based on gathering measured data, improving assembling processes and estimating model parameters via inverse analysis. More specifically, when dealing with viscoelastic materials, it is important to mention that the dynamic response of a structure can be influenced by uncertainties in the structure of the constitutive model used to describe its behaviour and also by uncertainties in constitutive model parameters. Moreover, simplifications and/or approximations in the constitutive and kinematic models may not lead to a full representation of the strain mechanisms. All of this generates uncertainties when computing model predictions of laminated structures with viscoelastic core [17].

To the authors' best knowledge, there is no paper addressing uncertainty quantification analysis for laminates containing viscoelastic layers described by fractional derivative constitutive models. The aim of the present paper is to get more robust models to be used for design and optimization. This paper performs the propagation of uncertainties, which were obtained out of an experimental-set up [18], in two different models for laminated structures with viscoelastic core, namely: Mead and Markus and a more complex model recently proposed by Arvin et al. [14]. The impact of uncertainties on modal parameters are analysed. As the influence of model parameters on modal parameters are ruled by nonlinear relationships, this uncertainty quantification analysis for viscoelastic laminate beam model are performed by Monte Carlo simulations. More specifically, the modelling uncertainties are considered to be associated with manufacturing process, assembling and disassembling components and constitutive viscoelastic model parameters.

In the following sections, the constitutive model formulation based on fractional derivative operators with five parameters is first described, and it is followed by the mathematical formulation for the two kinematic models. The models are numerically implemented through the Finite Element Method (FEM) and the computation of the modal parameters is performed by means of an iterative optimization process. The modelling of uncertainties is introduced to stablish the procedure to obtain statistical information from the random modal parameters, which are used in the uncertainty propagation analysis for the two models of viscoelastic sandwich beam. The analyses of four uncertainty sources related to the constitutive parameters and to the geometric dimensions are presented, and the respective comparisons between the kinematic models are discussed, followed by concluding remarks.

2. Constitutive model formulation

Fractional derivative equations have been used to represent the mechanical behaviour of several materials. As for viscoelastic materials, the use of fractional derivative equations has found many applications such as the ones presented in [19–27], to cite a few. The general form for 1-D constitutive models for a Viscoelastic Material (VEM) based on time domain fractional derivative operators can be expressed as follows [21,22]

$$\sigma(t) + b_1 \frac{d^{\beta_1}}{dt^{\beta_1}}\sigma(t) + b_2 \frac{d^{\beta_2}}{dt^{\beta_2}}\sigma(t) + \cdots + b_n \frac{d^{\beta_n}}{dt^{\beta_n}}\sigma(t) = a_0\varepsilon(t) + a_1 \frac{d^{\alpha_1}}{dt^{\alpha_1}}\varepsilon(t) + a_2 \frac{d^{\alpha_2}}{dt^{\alpha_2}}\varepsilon(t) + \cdots + a_m \frac{d^{\alpha_m}}{dt^{\alpha_m}}\varepsilon(t) \quad (1)$$

where $\{a_1, \ldots a_m\}$ and $\{b_1, \ldots b_n\}$ are constant parameters of the material model, t is the time, $0 < \beta_r \leqslant 1$ and $0 < \alpha_s \leqslant 1$ are the fractional derivative orders for the stress $\sigma(t)$ and strain $\varepsilon(t)$ fields, respectively. In case all parameters $\{\beta_r, \alpha_s\}$ are integer numbers, Eq. (1) corresponds to a conventional VEM model [28]. Concerning the fractional derivative operator d^{v}/dt^{v} used in Eq. (1), it is defined as follows [27]

$$\frac{d^{v}}{dt^{v}}[\upsilon(t)] \equiv \frac{1}{\Gamma(1-\gamma)} \frac{d}{dt} \int_0^t \frac{\upsilon(t-\tau)}{\tau^{v}} d\tau \quad (2)$$

where $\Gamma(\cdot)$ is the Gamma function and v is the order of the fractional derivative operator, $0 \leqslant v \leqslant 1$ [21]. Eq. (2) clearly presents the hereditary characteristic of the fractional operator due to the fact it uses the entire history of the function $\upsilon(\tau)$ to compute its fractional derivative $\frac{d^{v}}{dt^{v}}[\upsilon(t)]$ at current time t.

A detailed analysis of fractional constitutive models for viscoelasticity from the point of view of the thermodynamics of irreversible processes is presented by Lion [29]. Pritz [19,23] presents analyses for two constitutive models derived from Eq. (1). More specifically, Pritz [19,23] considers models defined by 4 and 5 constitutive parameters for which $\beta_r = 0$ for $r \geqslant 3$ and $\alpha_s = 0$ for $s \geqslant 2$. Concerning the constitutive model with five parameters [19], it is able to provide an asymmetric loss factor peak and approximately constant loss factor at high frequencies. This model is presented in Eq. (3)

$$\sigma(t) + \tau^{\beta} \frac{d^{\beta}}{dt^{\beta}}\sigma(t) = G_0\varepsilon(t) + G_0\tau^{\beta}\frac{d^{\beta}}{dt^{\beta}}\varepsilon(t) + (G_\infty - G_0)\tau^{\alpha}\frac{d^{\alpha}}{dt^{\alpha}}\varepsilon(t) \quad (3)$$

where τ is the relaxation time, and G_0 and G_∞ are the static and dynamic modules, respectively. The model obtained when $\alpha = \beta$ in Eq. (3) corresponds to the fractional Zener model [19,27]. Furthermore, in case $\alpha = \beta = 1$, the constitutive model defined by Eq. (3) yields the classical Zener model [27].

The complex modulus $\tilde{G}(j\omega)$ for the constitutive model in Eq. (3) can be derived by the application of the Fourier transform to both sides of Eq. (3) and recasting it as follows

$$\tilde{\sigma}(j\omega) = G_0\left(1 + (d-1)\frac{(j\omega\tau)^{\alpha}}{1 + (j\omega\tau)^{\beta}}\right)\tilde{\varepsilon}(j\omega) = \tilde{G}(j\omega)\tilde{\varepsilon}(j\omega) \quad (4)$$

where $d = G_\infty/G_0$ is the ratio between the dynamic and static modulus, $\tilde{V}(j\omega)$ denotes the Fourier transform of the function $\upsilon(t)$, ω is the circular frequency in rad/sec and $j = \sqrt{-1}$ is the imaginary number. The complex modulus is a frequency dependent operator and its constitutive parameters $\boldsymbol{\theta} = \{G_0, d, \tau, \alpha, \beta\}$ should be obtained by appropriate inverse analysis [18,30,31].

3. Mathematical models for laminated structures with viscoelastic core

Although commercial softwares may provide refined discrete models for this type of structural component, one may find in the literature several finite element models to describe the physical behaviour of laminates with a viscoelastic core sandwiched between elastic layers. This is inherently associated to the fact that an increase in model refinement is linked to an increase in computational costs. Furthermore, this type of component is in general a subsystem of a larger structural system that has to be controlled or optimised and whenever possible the Engineering team should use low order models that are still accurate enough for the applications of interest. Several authors proposed models to describe the dynamic behaviour of this type of viscoelastic laminate structural component such as Mead and Markus [2], Bai and Sun [3], Babber et al. [32], Chen and Chan [33], Arvin et al. [14], Won et al. [34], to cite a few.

Let us consider a slender multilayer laminate structure $\mathcal{B}$ occupying the region $[0, L] \times [0, b_h] \times [0, b_w] \subset \Re^3$ and composed of a viscoelastic core sandwiched between the two elastic layers. This