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# A multi-physics and multi-time scale approach for modeling fluid–solid interaction and heat transfer

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ABSTRACT

A novel non-conservative formulation for equations governing thermo-mechanical phenomena is developed to address multi-material and multi-physics issues. The first key point is that this formulation achieves a unifying equation for compressible viscous fluid flow and elastic solid deformation. The second is that the thermo-mechanical equations are both written with velocity and thermal flux variables to solve them simultaneously. With that formulation, interaction conditions at the fluid–structure interface become implicit and state equation is no longer necessary. Multi-time scale problems are solved, from the time scale of acoustic and thermoacoustic wave propagation, to longer time scale of fluid flow and thermal diffusion.



## 1. Introduction

Many physical phenomena in materials science, biomedecine or process engineering are dependent on compressible or incompressible fluid mechanics, acoustics or non-linear wave propagation [1], heat transfer, elastic solid mechanics. These physical phenomena are very often coupled leading to complex problems such as solid particle behavior in a fluid flow [2,3], plasma flow [4], fluid–structure interaction for aerospace [5,6], or biomedical applications [7–10], thermo-elastic fracture [11], transmission and reflection of elastic waves through an interface between dissimilar materials [12], thermoelastic shock wave diffraction with a multimaterial interface where fluid compressibility cannot be ignored [6,13–16].

The usual approaches for modeling such multi-physics problems are based on the standard equations that govern materials that often differ with the time scale considered. For instance, for short time scale characteristics in two-phase flows, thermal energy and material compressibility are considered through the Euler equations, whereas isothermal incompressible viscous flow is considered through Navier–Stokes equation for longer time scale steady-state flows. Similarly, for temperature step variation, Fourier's equation and a compressible form of the Navier–Stokes equations are used to model thermoacoustic waves propagation. When a large discrepancy between the time scale constants of the coupled governing equations is present, for instance that of sound velocity and thermal diffusion, solving process needs to consider the time increment consistent with the lower time constant to avoid numerical instabilities, thus leading to time-consuming computation. Thus, for slow heating involving large time scale modeling, incompressible Navier–Stokes and heat transport equations are used since fluid flow convection and thermal diffusion time constants have closer order of magnitude.

Then, when the mechanical governing equations of the materials involved are not closely related which is the case for a fluid and an elastic solid, interface coupling also implies additional boundary equations at the interface. Simulation of such fluid–structure interaction problems called as fluid–structure interaction can be addressed through numerical methodologies which sequentially solve each governing equation with data transfer through the boundary conditions which can be temperature, pressure, shear stress or interface velocity depending on the problem [6,16–22]. Some other approaches introduce a pseudo-simultaneous solving process for velocity and displacement fields in the fluid and solid domains, respectively, by implementing implicit iterative sequential coupling in order to converge at any time step and thus to avoid non-physical interface oscillations [8,10]. This latter approach is defined as monolithic in contrast to the previous partitioned ones, but does not strictly solve system variables

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simultaneously, which will insure accuracy and stability of the interface coupling. The Arbitrary Lagrangian Eulerian (ALE) [5,7,14,23–28] method which can be coupled to both monolithic and partitioned approaches has the advantage of discretizing strictly separated material. It can be used for viscous flow governed by Navier–Stokes equation as well as elastic solid equation. This method based on a moving mesh lead nevertheless to distorted elements giving rise to re-meshing developments for efficient calculations [26]. Finite elements methods have been largely improved in order to increase the convergence [29,30] or to enhance the stress prediction [31]. On the opposite, Eulerian method based on a cartesian grid leads to a non-conforming interface, which has given rise to many numerical developments either within partitioned [22,15] or monolithic [32,33] procedures in order to increase the accuracy of coupling the two materials at the interface. To overcome such inherent limitations of the numerical methods based on moving meshes and non-conforming interfaces, the method of finite spheres was recently developed [34,35] and applied to elastic wave propagation problems [36].

Although numerical method developments or improvements are essential to deal with complex multi-material behaviors involving coupled governing equations, the development of new mathematical formulations is essential as well in order to avoid mismatch between governing equations for short and long time scale modeling, and to obtain strong fluid–structure coupling at the interface by solving strictly simultaneously the governing equations. An advance with such mathematical formulations was made in the particular case of isothermal multi-fluid systems involving either compressible or incompressible flows [37] and for the modeling of trans-critical path from supercritical to subcritical states [38]. In this paper we aim to develop the complete mathematical formulation for compressible and incompressible viscous flow, elastic solid deformation and heat transfer allowing to solve simultaneously for the velocity, displacement and temperature fields of the multi-material system. More precisely, this formulation has to combine the six main points discussed below.

- Compressible and "incompressible" fluids and solids are characterized by physical constants which are not strictly related, for instance Lamé's coefficients of an elastic solid and that of a fluid. The first point is thus to unify the material intrinsic properties which are involved in the conservative equations in order to reduce the different mechanical equations to a unique equation for strong coupling across the interface. This implies that every fluid will have to be considered as compressible or almost incompressible, in relation to its experimental thermodynamic coefficient.
- The second point is to be able to formulate the unified mechanical equation and the thermal equation with two vectorial variables in order to be able to solve for them simultaneously through a single linear system.
- The third point concerns time scaling. Obviously, it is the shorter time constants of the physical phenomena involved (acoustic, dynamic or thermal) that have to be considered. But, the formulation will have to give the possibility to obtain the solution using a higher time constant, allowing continuous evolution from a compressible fluid at the time scale of wave propagation to an incompressible fluid at the time scale of steady state evolution.
- As pointed out previously, the model has to deal with compressible fluids. In the literature highly compressible phenomena are resolved through a conservative formulation where the momentum $\rho\mathbf{V}$, may result in different types of discontinuities for density $\rho$ and velocity $\mathbf{V}$. While it is consistent to choose $\rho\mathbf{V}$ as a variable when it varies slightly as for one-phase flows, this is no longer the case for two-phase flows where one phase is highly compressible and the other not. Indeed density can vary greatly across the interface whereas velocity varies continuously and slightly. So, the third point is to formulate a model with the velocity vector as variable, density being related to mass flux. Studies dealing with strongly two-phase shocked flows in the framework of a conservative formulation [13] prove to be insufficiently accurate to deal with two-phase continuous flows.
- If density is related to mass flux, we can discuss the suitability of using the state equation to determine density knowing temperature and pressure. Numerous numerical studies use state equations formulated to be valid over a broad interval of variable variation, particularly the pressure. Even in the case for the stiffened equation of state [15,39,40] non-physical variable smoothing at the interface is obtained. The fourth point is thus to take the counterpart to the use of state equations by considering the thermodynamic coefficients. They can be more easily obtained from experimental data tables and semi-empirical laws and will not modify the formulation of the mathematical model as using the state equations does.
- Finally, the mathematical model proposed has to be independent of the numerical methodology allowing the use of either moving mesh or ALE methods where the different materials are strictly separated or Cartesian grids where interface tracking methods can be Lagrangian [41–43] or Eulerian [44,45].

## 2. Unifying model for fluid flow and elastic solid deformation coupled with heat transfer

Let us consider a two-phase domain $\Omega$ delimited by a surface $\Gamma$. The interface between the two phases is noted $\Sigma$ (Fig. 1). These two phases can be either isotropic elastic solids or Newtonian viscous fluids. The case where one phase is an elastic solid and the other a Newtonian viscous fluid is considered in the model as well. No mass exchange through the interface is involved, leading to a divariant system.

The thermodynamic state can thus be described at any time $t$ and any material point $M$ with two intensive variables, for instance the absolute temperature $T(M)$ and the pressure $p(M)$. Density $\rho(M)$ and every thermodynamic coefficient such as specific heat $c_v(M)$, isothermal compressibility coefficient $\chi_T(M)$, constant volume compressibility coefficient $\alpha(M)$ and coefficient of thermal expansion $\beta(M)$ are thus entirely defined from these two variables. These thermodynamic coefficients are defined below together with the relationship between them Eq. (1). All these coefficients can be determined experimentally whatever the method used.

$$\begin{cases} \alpha = \frac{1}{p}\left(\frac{\partial p}{\partial T}\right)_\rho & \beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p & \chi_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_T \\ c_p = \left(\frac{\partial h}{\partial T}\right)_p & c_v = \left(\frac{\partial e}{\partial T}\right)_\rho & \\ -\left(\frac{\partial \rho}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_\rho \left(\frac{\partial p}{\partial \rho}\right)_T = \frac{\beta}{\alpha p \chi_T} = 1 & & \end{cases} \quad (1)$$

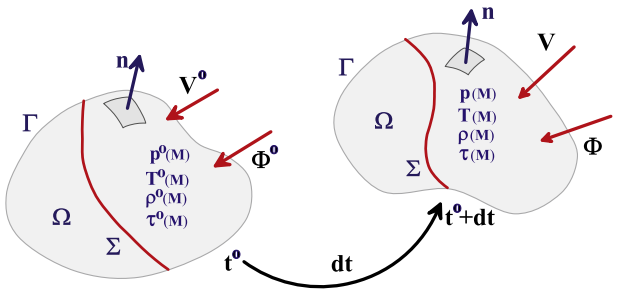


**Fig. 1.** Evolution of the Lagrangian thermo-mechanical variables for a material point $M$ between time $t^0$ to time $t = t^0 + dt$.