Computers and Structures 164 (2016) 63–74

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Optimization of hardening/softening behavior of plane frame structures using nonlinear normal modes

Suguang Dou [a,*], Jakob Søndergaard Jensen [b]

[a] Department of Mechanical Engineering, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark
[b] Department of Electrical Engineering, Technical University of Denmark, 2800 Kgs. Lyngby, Denmark

ARTICLE INFO

Article history:
Received 28 May 2015
Accepted 2 November 2015
Available online 1 December 2015

Keywords:
Finite element
Structural optimization
Nonlinear normal mode
Harmonic balance
Adjoint method
Hardening/softening behavior

ABSTRACT

Devices that exploit essential nonlinear behavior such as hardening/softening and inter-modal coupling effects are increasingly used in engineering and fundamental studies. Based on nonlinear normal modes, we present a gradient-based structural optimization method for tailoring the hardening/softening behavior of nonlinear mechanical systems. The iterative optimization procedure consists of calculation of nonlinear normal modes, solving an adjoint equation system for sensitivity analysis and an update of design variables using a mathematical programming tool. We demonstrate the method with examples involving plane frame structures where the hardening/softening behavior is qualitatively and quantitatively tuned by simple changes in the geometry of the structures.

1. Introduction

Linear normal modes (LNMs) play a significant role in design optimization of mechanical components for their dynamic response. When applied to nonlinear mechanical systems, this may lead to sub-optimal designs because of the unresolved nonlinear behavior such as frequency-energy dependence and internal resonances. Moreover, a variety of novel applications that exploit essentially nonlinear behavior are increasingly used, e.g. in engineering for vibration mitigation [1–3], and particularly in nonlinear micromechanical and nanomechanical resonators for energy harvesting [4], frequency stabilization [5,6], frequency division [7], etc. For a comprehensive review of nonlinear micro- and nanoresonators, the reader may consult [8,9]. Since structural optimization of these inherently nonlinear devices cannot be fully achieved using linear structural dynamics, it is of considerable value to develop efficient techniques for structural optimization based on nonlinear structural dynamics.

As a nonlinear analog of LNMs, nonlinear normal modes (NNMs) provide a systematic way to investigate the nonlinear behavior in nonlinear structural dynamics, particularly frequency-energy dependence and modal interaction [10]. The resulting NNMs also provide valuable insight into the damped system. For example, the temporal evolution of the instantaneous frequencies in free decay responses follows NNMs of the undamped system. Based on this feature, free decay tests in conjunction with continuous wavelet transform have been used in experimental analysis of NNMs [11]. Further, the nonlinear forced resonances are in the neighborhood of NNMs. This feature is of practical importance in engineering design, e.g., dynamic tests of NNMs in forced vibrations [12] and efficient optimization of nonlinear resonance peaks [13–15]. Computation of NNMs using numerical methods [16], particularly the harmonic balance method [17–19], is especially attractive for their abilities to be combined with nonlinear finite element models, which facilitate element-based design parameterization of structural geometry and further application of advanced structural optimization (e.g. shape and topology optimization) [20]. In a previous study, based on the normal forms linked to NNMs [21,22], we studied structural optimization of hardening/softening behavior and nonlinear modal coupling effects [23], where the analysis and optimization is limited to mechanical systems with polynomial-type nonlinearity.

In this paper, we extend the current structural optimization procedure to the more general case of modal analysis of nonlinear mechanical systems. For general applicability, NNMs are numerically computed using the harmonic balance method [13] in conjunction with the alternating frequency/time domain method (AFT) [24], which has the ability to handle complex nonlinearities. The analysis method is applicable to NNMs of conservative systems, such as nonlinear systems with geometric nonlinearity and polynomial-type material nonlinearity. For NNMs of

* Corresponding author.
E-mail addresses: sudou@mek.dtu.dk (S. Dou), json@elektro.dtu.dk (J.S. Jensen).

http://dx.doi.org/10.1016/j.compstruc.2015.11.001
0045-7949/

nonconservative systems such as nonlinear systems with friction and hysteretic material nonlinearities, the reader can refer to [25]. Based on the resulting NNMs, we propose a gradient-based structural optimization formulation for tailoring hardening/softening behavior. For a relatively comprehensive review of the hardening/softening behavior, the reader can refer to [22,26]. Examples involving plane frame structures are used to demonstrate that both quantitative and qualitative tuning of hardening/softening behavior (e.g. from softening behavior to hardening behavior, and vice versa) can be achieved by a simple manipulation of the structural geometry. In the analysis, these structures are modeled with two-dimensional beam elements based on the geometrically exact theory [27,28], which does not make any approximation of the trigonometric functions arising in kinematic relations and hence can be used for vibrations with large in-plane motion. Since NNM is a nonlinear analogy of LNM, the proposed optimization methodology can also be viewed as an extension of the optimization of eigenvalue problems in linear structural dynamics (e.g., eigenvalues and eigenvectors) [29–32].

The article is organized as follows. First, computation of NNM using the incremental harmonic balance (IHB) in conjunction with the alternating frequency/time domain method is presented in Section 2. In Section 3, a general optimization problem for tuning NNM with hardening/softening behavior and its sensitivity analysis are formulated. Optimization examples for tailoring the hardening/softening behavior of plane frame structures are presented in Section 4 and conclusions are drawn in Section 5.

2. Nonlinear modal analysis

An ideal starting point for numerical computation of NNM is continuation of periodic responses in the neighborhood of a LNM. In this case, the NNM reduces to a LNM when the vibration amplitude is sufficiently small. In computation of the NNM for plane frame structures, IHB is applied to a nonlinear finite element model. In contrast to the time marching method widely used in nonlinear structural dynamics, IHB is an efficient way to compute the time-periodic response by representing it in Fourier series and solving a set of nonlinear algebraic equations in terms of the Fourier coefficients and the frequency. In the continuation, with a given initial guess or the responses in previous steps, the unknown response in the neighborhood is first predicted and then iteratively corrected.

2.1. Harmonic balance method

The equation of motion of a discretized undamped mechanical system (e.g., a nonlinear finite element model without dissipation and load) is assumed as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{g}(\mathbf{q}(t)) = \mathbf{0} \tag{1}$$

where $\mathbf{q}(t)$ denotes time-periodic response to be computed, $\mathbf{M}$ is the symmetric and positive definite mass matrix, and $\mathbf{g}(\mathbf{q})$ denotes the nonlinear stiffness force. Further, the differentiation of $\mathbf{g}(\mathbf{q})$ with respect to $\mathbf{q}$, denoted as $\mathbf{K}(\mathbf{q})$, is the symmetric tangent stiffness matrix.

First, a new time scale τ is introduced as $\tau = \omega t$, where ω denotes the fundamental frequency of the response in radians per second. So the equation of motion in Eq. (1) can be rewritten as

$$\omega^2 \mathbf{M}\mathbf{q}''(\tau) + \mathbf{g}(\mathbf{q}(\tau)) = \mathbf{0} \tag{2}$$

where the prime indicates the differentiation with respect to the new time scale τ. It is noted that with the new time scale, the period of the system response is normalized to 2π, and thereby the derivation and numerical implementation does not require the real value of the period of the response, which can be calculated as $T = 2\pi/\omega$ when ω is given. Then the system response is expanded into Fourier series. For the displacement of one degree of freedom, e.g. $q_i(\tau)$, its Fourier series is expressed as

$$q_i(\tau) = a_{i0} + \sum_{n=1}^{N_H} (a_{in}\cos(n\tau) + b_{in}\sin(n\tau)) \tag{3}$$

where i denotes the ith degree of freedom in the finite element model, n denotes the nth-order harmonic and N_H is the highest order of retained harmonics. Generally, the value of N_H can be decided by performing a convergence study of the solution with respect to N_H. A guideline for selecting the value of N_H based on two conditions: completeness and balanceability, can be found in [33,34]. In matrix form, Eq. (3) is written as

$$q_i(\tau) = \mathbf{s}\overline{\mathbf{q}}_i \tag{4}$$

where $\mathbf{s}$ and $\overline{\mathbf{q}}_i$ represent the Fourier basis and the retained Fourier coefficients of $q_i(\tau)$, defined as

$$\begin{aligned} \mathbf{s} &= [\,1 \quad \cos\tau \quad \cdots \quad \cos(N_H\tau) \quad \sin\tau \quad \cdots \quad \sin(N_H\tau)\,] \\ \overline{\mathbf{q}}_i &= [\,a_{i0} \quad a_{i1} \quad \cdots \quad a_{iN_H} \quad b_{i1} \quad \cdots \quad b_{iN_H}\,]^T \end{aligned} \tag{5}$$

It is noted that $\mathbf{s}$ is of dimension $1 \times 2(N_H+1)$ and $\overline{\mathbf{q}}_i$ is of dimension $2(N_H+1) \times 1$. Based on the Fourier series of the displacement of one degree of freedom, the Fourier series of the displacements of all degrees of freedom are now written as

$$\mathbf{q}(\tau) = \mathbf{S}\overline{\mathbf{q}} \tag{6}$$

where $\mathbf{q}(\tau)$ of dimension $N_D \times 1$ represents the displacements with N_D being the number of degrees of freedom in the model, $\mathbf{S}$ of dimension $N_D \times (N_D(2N_H+1))$ represents a set of Fourier basis, and $\overline{\mathbf{q}}$ of dimension $(N_D(2N_H+1)) \times 1$ represents the corresponding Fourier coefficients by projecting $\mathbf{q}(\tau)$ onto $\mathbf{S}$, i.e.

$$\mathbf{S} = \mathbf{diag}(\mathbf{s}, \ldots, \mathbf{s}) = \begin{bmatrix} \mathbf{s} & & \\ & \ddots & \\ & & \mathbf{s} \end{bmatrix}, \quad \overline{\mathbf{q}} = [\,\overline{\mathbf{q}}_1^T \quad \cdots \quad \overline{\mathbf{q}}_{N_D}^T\,]^T \tag{7}$$

To determine the Fourier coefficients and the frequency of the response, the Galerkin method is applied to the ordinary differential equation in Eq. (2). By substituting Eq. (6) into Eq. (2), multiplying Eq. (2) with $\mathbf{S}^T$ and performing integration with respect to τ on $[0, 2\pi]$, an algebraic equation system is obtained as

$$\omega^2 \overline{\mathbf{M}}\overline{\mathbf{q}} + \overline{\mathbf{g}} = \mathbf{0} \tag{8}$$

where the barred terms $\overline{\mathbf{M}}$ and $\overline{\mathbf{g}}$ are given as

$$\overline{\mathbf{M}} = \frac{1}{2\pi}\int_0^{2\pi} \mathbf{S}^T\mathbf{M}\mathbf{S}''\,\mathrm{d}\tau, \quad \overline{\mathbf{g}} = \frac{1}{2\pi}\int_0^{2\pi} \mathbf{S}^T\mathbf{g}(\mathbf{q}(\tau))\,\mathrm{d}\tau \tag{9}$$

It is noted that Eq. (8) can be viewed as the frequency-domain counterpart of the equation of motion in Eq. (1), and $\overline{\mathbf{M}}$ and $\overline{\mathbf{g}}$ can be interpreted as the frequency-domain counterparts of $\mathbf{M}$ and $\mathbf{g}$, respectively.

Since the algebraic equation system in Eq. (8) is nonlinear, we use an iterative Newton–Raphson method to solve it. The procedure of numerical implementation is described later in the continuation approaches, whereas the incremental form of Eq. (8) is given here as

$$(\omega^2\overline{\mathbf{M}} + \overline{\mathbf{K}})\Delta\overline{\mathbf{q}} + (2\omega\overline{\mathbf{M}}\overline{\mathbf{q}})\Delta\omega = -(\omega^2\overline{\mathbf{M}}\overline{\mathbf{q}} + \overline{\mathbf{g}}) \tag{10}$$

where $\overline{\mathbf{K}}$ is the frequency-domain counterpart of the tangent stiffness matrix $\mathbf{K}$, and this relation is derived as

$$\overline{\mathbf{K}} = \frac{1}{2\pi}\int_0^{2\pi} \mathbf{S}^T \frac{\partial\mathbf{g}(\mathbf{q}(\tau))}{\partial\mathbf{q}(\tau)}\frac{\partial\mathbf{q}(\tau)}{\partial\overline{\mathbf{q}}}\,\mathrm{d}\tau = \frac{1}{2\pi}\int_0^{2\pi} \mathbf{S}^T\mathbf{K}(\mathbf{q}(\tau))\mathbf{S}\,\mathrm{d}\tau \tag{11}$$