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Design sensitivity analysis of structures with viscoelastic dampers

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ABSTRACT

In paper the design sensitivity analysis of dynamic characteristics of structures with viscoelastic (VE) dampers is considered. The dampers are modeled using the classical rheological model and the rheological model with fractional derivatives. The design sensitivity is analyzed by using direct differentiation method and adjoint variables method. The formulae enabling the calculation of sensitivity of the first and the second order with respect to chosen design parameters have been derived. Sensitivity analysis of eigenvalues, eigenvectors and dynamic characteristics such as natural frequencies and nondimensional damping ratios are presented. The correctness of the presented formulae is illustrated in examples.

1. Introduction

Design sensitivity analysis quantifies the influence of changes of parameters of structures or systems on the quantities which measure their performance. The design sensitivity analysis of structures and mechanical systems is a very important part of the solution procedure in many engineering problems such as optimization of structures, parametric identification, structural health monitoring, model updating [1], structural reliability, damage detection [2] and other ones. The concept of stochastic sensitivity was proposed by Szopa in [3]. Recently, the idea of the so-called global sensitivity analysis has been developed in [4]. In paper [1], the authors pointed out that the sensitivity-based method is probably most successful in the model updating, in comparison with other ones.

Sensitivity methods and problems are reviewed, for example in [5]. Computational methods for sensitivity analysis, particularly those related to the eigenvalue problems, have drawn much attention for the last few decades. In general, methods for calculating the design sensitivity of structures could be divided into semi-analytic methods, direct sensitivity method [2,6] and adjoint variables methods [7,8]. The design sensitivity of eigenvalues and eigenvectors of undamped structures and systems is considered in many papers (see, for example [1,9]). Methods for the sensitivity analysis of systems or structures with proportional and/or nonproportional damping are presented in [6,10–13]. Both systems with distinct and repeated eigenvalues are considered in [9–11]. The sensitivity analysis for general nonlinear eigenproblem is considered in [12]. The methods of design sensitivity analysis could also be divided into two groups: (i) the algebraic method applied in determination of sensitivities of eigenvalues, eigenvectors and the frequency response function (FRF) and (ii) the modal approach applied mainly to determine sensitivities of eigenvectors and frequency response functions. The first-group methods are presented in [2,6,8–13] while the modal approach is described in [13,14]. Design sensitivity analysis of the frequency response functions is considered by Choi and Lee [15], and in the papers by Ting [16] and by Qu and Selvam [17]. The above-mentioned studies consider only viscously damped structures or systems.

The sensitivity analysis of eigenvalues and eigenvectors of nonviscously damped systems was presented by Adhikari [14,18], Adhikari and Friswell [19], and Li et al. [11,12,20]. The sensitivity analysis of dynamic response of nonviscously damped systems is considered by Li et al. [13], who described damping forces by means of the convolution integral resulting from the Boltzmann's superposition principle. The sensitivity analysis of systems with damping, described with the help of fractional derivatives is presented by Kobelev [21]. The method described in [13], in principle, could also be used to determine the parametric sensitivity of nonviscously damped systems when the fractional derivatives are used to describe damping forces. However, the results of sensitivity analysis of structures with fractional dampers are not presented. In the context of dynamics of viscoelastic structures, eigensensitivity analysis is presented in [22,23], where the concept of complex modulus is used to describe the viscoelastic properties of surface damping layers. Additionally, in [23] the sensitivity of frequency response functions with respect to changes of temperature is considered.

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The systematic design sensitivity analysis of structures with viscoelastic (VE) dampers which is referred to in this work has never been published before. In this paper, the direct differentiation method for the design sensitivity analysis of structure with VE dampers is discussed. The systematic approach for the sensitivity analysis of dynamic characteristic of structures with dampers is described. These include the sensitivity of natural frequencies, non-dimensional damping ratios, sensitivities of eigenvalues and eigenvectors. Both the direct approach and the adjoint approach are presented. The dampers are modeled using both the classical rheological model and the rheological model with fractional derivatives.

The paper is organized as follows. In Section 2, the considered models of damper and the equation of motion of structure with dampers are presented. Then, in Section 3, the method to calculate the dynamic characteristics of systems is briefly described. Next, in Section 4, the design sensitivity analysis is shown. In Section 5, the results of exemplary calculations are provided and correctness of the method used is discussed. Finally, in Section 6, concluding remarks are formulated. Several useful formulae are given in the Appendices.

2. Model of VE dampers and equation of motion of structures with VE dampers

2.1. Models of VE dampers

It is well-known that the properties of VE materials which are used to build VE dampers depend on many parameters such as: frequency of excitation, temperature, and amplitudes of vibration [24]. The influence of the amplitudes of vibration is neglected in the case of small amplitudes. Moreover, the effect of temperature is, usually, also skipped and finally only the influence of excitation frequency is taken into account.

Several models are used to describe the dynamic behavior of VE materials and VE damper. In general, these can be divided into three groups, i.e., rheological (mechanical) models, phenomenological models, and other ones. Mainly, the rheological models, both the classical one [25–31] and the fractional derivative model [32–38] are considered.

In description of structure with VE materials in time domain three models are relatively widely used: Golla–Hughes–McTavish model (GHM) considered in [48,49], augmenting thermodynamic fields model (ATF) considered in [50] and anelastic displacement fields model (ADF) considered in [51]. These models are applied when internal variables approach are taken into consideration but to description of VE dampers have not been used.

In this paper, only the classical rheological models and ones described with the help of fractional derivatives are taken into account.

The diagram of the considered classical model is shown in Fig. 1 while in Fig. 2, diagram of fractional model is presented.

In the case of the fractional model of damper, the dashpot is replaced by a viscoelastic element (called also the Scott-Blair element) described by two parameters (in Fig. 2 shown as a rhombus), the constant c and a number α $(0 < \alpha \leqslant 1)$.

The constitutive equation for the Scott-Blair element is:

$$u(t) = c\, D_t^{\alpha} \Delta q(t), \tag{1}$$

where the symbol $D_t^{\alpha}(\bullet)$ denotes the fractional-derivative of the order α with respect to the time t. Mainly the Riemann–Liouville definition of fractional derivative is used in the description of VE dampers and in rheology [28,32–40]. However, recently Di Paola et al. [41,42] have shown that, in the context of rheology, it is more logical to use the Caputo type definition of fractional derivative.

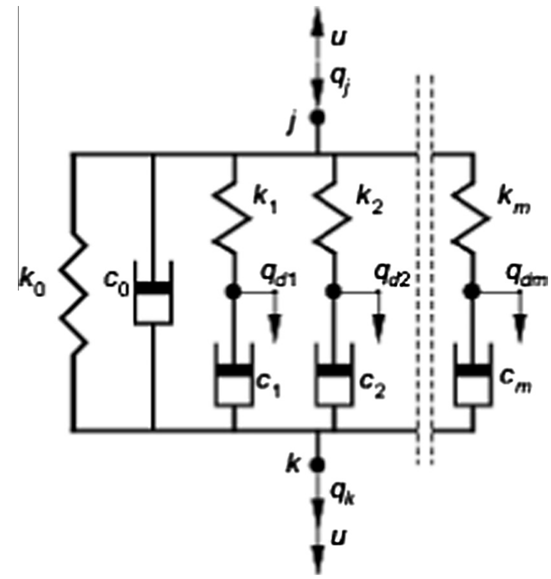

Fig. 1. The diagram of the generalized classic Maxwell model of damper.

Moreover, it is known (see [43]) that for a system at rest at $t = 0$ or for systems that operate from $t = -\infty$, the Caputo fractional derivative is equivalent with the Riemann–Liouville derivative. In conclusion, when the above assumptions are fulfilled, the operator $D_t^{\alpha}(\bullet)$ could be understood in two ways: as the Riemann–Liouville derivative or as the Caputo derivative.

The fractional model of damper that is adopted in this paper is shown in Fig. 2. It consists of the fractional Kelvin element which is connected in parallel with the fractional Maxwell element. The total force $u(t)$ in this model is the sum of forces that occur in the Kelvin element:

$$u_0(t) = k_0 \Delta q(t) + c_0 D_t^{\alpha} \Delta q(t), \tag{2}$$

and the force which acts in the fractional Maxwell element and is governed by the equation:

$$\nu_1 u_1(t) + D_t^{\alpha} u_1(t) = k_1 D_t^{\alpha} \Delta q(t), \tag{3}$$

where $\nu_1 = k_1/c_1$, $\Delta q(t) = q_k(t) - q_j(t)$ is the relative displacement of damper.

Applying the Laplace transform with zero initial conditions, to equations of motion (2) and (3) the following Laplace transform of total force $\bar{u}(s)$ is obtained:

$$\bar{u}(s) = \left(k_0 + s^{\alpha} c_0 + \frac{k_1 s^{\alpha}}{\nu_1 + s^{\alpha}} \right) \Delta\bar{q}(s) = G(s) \Delta\bar{q}(s), \tag{4}$$

where such quantities as $\bar{u}(s)$ and $\Delta\bar{q}(s)$ denote the Laplace transforms of $u(t)$ and $\Delta q(t)$, respectively, and s is the Laplace variable.

The fractional models of damper described above can be treated as general ones. A set of specific models arise from them: the simple fractional Maxwell (when $k_0 = c_0 = 0$), the fractional Kelvin model (when $k_1 = \nu_1 = 0$) and the fractional Zener model (when $c_0 = 0$). This means that almost all fractional models known from literature up to now are taken into account by the above fractional model.

The generalized classic Maxwell model of damper is built of a dashpot with the constant c_0, connected in parallel with a spring of the stiffness k_0 and, in addition to them, a number of two-parameter Maxwell elements which are connected in parallel.

The Maxwell elements are characterized by the stiffness and damping parameters k_l and c_l $(l = 1, 2, \ldots, m)$. Moreover, the part of the model that is associated with the constants c_0, k_0 constitutes