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An improved fully stressed design evolution strategy for layout optimization of truss structures

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ABSTRACT

During the recent decade, truss optimization by meta-heuristics has gradually replaced deterministic and optimality criteria-based methods. While they may provide some advantages regarding their robustness and ability to avoid local minima, the required evaluation budget grows fast when the number of design variables is increased. This practically limits the size of the problems to which they can be applied. Furthermore, many recent stochastic optimization methods handle the size optimization only, the potential saving from which is highly limited, when compared to the most sophisticated, and obviously the most challenging scenario, simultaneous topology, shape and size (TSS) optimization. In a recent study by the authors, a method based on combination of optimality criteria and evolution strategies, called fully stressed design based on evolution strategies (FSD-ES), was proposed for TSS optimization of truss structures. FSD-ES outperformed available truss optimizers in the literature, both in efficiency and robustness. The contribution of this study is twofold. First, an improved version of FSD-ES method, called FSD-ES-II, is proposed. In comparison with the earlier version, it takes the displacement constraints in the resizing step into account and can handle constraints governed by practically used specifications. Update of strategy parameters is also revised following contemporary and new developments in evolution strategies. Second, a test suite involving a number of complicated TSS optimization problems is chosen to overcome usual shortcomings in the available benchmark problems. For each problem, performance of FSD-ES-II is compared with the best results available in the literature, often showing a significant superiority of the proposed approach.

1. Introduction

Truss design optimization can be considered from three distinct perspectives. Topology optimization determines the optimum connection plot of members. Shape optimization optimize coordinates of the nodes for a known topology and size optimization finds the optimum cross sections of the members. The optimized structure should satisfy some constraints on member stress, node displacement, slenderness ratio or even natural frequency.

The methods commonly used for truss optimization are based on optimality criteria or mathematical programing [1,2]. The former assumes that the optimal design should satisfy some *a priori* conditions [3,4]. The concept of fully stressed design (FSD) is the most common approach in this group, which assumes in the optimally sized structure, all members reach the stress limit at least in one of the load cases [3]. Accordingly, all members are iteratively resized to reach this goal, assuming that the force distribution does not change when members are resized. These assumptions are not always valid. First, the global minimum in not necessarily a fully stressed design [4,2]. Second, member forces change as soon as their section area is modified, except in determinate structures, in which FSD can potentially converge in one iteration. Nevertheless, when the number of redundant members is small, the error prompted by these assumptions is usually small, and iterative resizing, at least when the resizing step is controlled [3], can reach a high quality design. The required number of design evaluations is almost independent of the number of members [3], and the method usually reaches a good solution after a few iterations [3]. Later, the concept of FSD was extended to handle problems with multiple load cases, displacement constraints, or when more sophisticated failure criteria are governed by design specifications [1].

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For these reasons, FSD used to be preferred over mathematical programming, when the computation resources were limited [3], except for highly indeterminate structures, where FSD risks divergence [3]. However, it does not take the objective function into account and thus, use of more sophisticated objective functions that consider other factors in the overall cost, is not directly applicable. When there are multiple displacement constraints, FSD leads to a resizing problem which is not easy to solve analytically.

Unlike optimality criteria, mathematical programming methods are robust tools to solve general optimization problems [3]. With recent development in computation tools and parallel computing, the challenge of costly evaluations has been moderated to great extent. At the same time, stochastic optimization techniques such as evolutionary algorithms (EAs) and swarm-based methods were developed and demonstrated some advantages over deterministic approaches, especially in multimodal problems. There has been a large number of studies on truss optimization with stochastic methods in the recent decade. Many of them, however, consider the simplest scenario, size optimization. The number of recent publications on size optimization with stochastic methods is relatively huge, too many to cite. Some examples of size optimization methods and a review of meta-heuristics applied to truss optimization can be found in previous publications [5–8].

More sophisticated schemes consider shape or topology optimization as well [9–11,8]. Topology optimization is particularly a challenging task, since even a small variation in topology can result in significant change in member forces and besides, many kinematically unstable structures might be produced during the search. The most sophisticated scheme, and potentially the most effective one [2], performs topology, shape and size (TSS) optimization at the same time. Nevertheless, studies on TSS optimization are comparatively scarce, possibly because of the complexity of the problem nature which demands sophisticated specialization of meta-heuristics. Several strategies to circumvent this complexity, in the case of TSS optimization, were proposed in the literature, however, they usually reduces potential for better solutions [12]. Moreover, the size of the test problems employed to validate the algorithms is usually small or moderate at best [13–18]. A few studies tried fairly large problems as well [19,20,12], but a comparison with other methods was not provided.

In a recent method, called fully stressed design based evolution strategy (FSD-ES) [11,12], the concept of FSD was employed to resize the designs produced by an evolution strategy. In comparison with earlier stochastic optimization methods, FSD-ES could reach lighter structures in smaller number of function evaluations. The resizing step helps the method find near optimally sized structure for a given shape or topology defined by the evolution strategy.

This study aims at overcoming some general drawbacks in stochastic TSS truss optimization by improving the earlier version of FSD-ES. The contributions of this study to truss optimization field are as follows:

- An improved version of the resizing technique is proposed, which can take displacement constraints into account. We also propose an optimality criteria-based heuristic to solve the resizing problem.
- The employed evolution strategy is revised and the traditional mutative self-adaptation concept is replaced by a strategy based on contemporary evolution strategies.
- A revision to the fitness function is provided to compare kinematically unstable structures as well. The problem of under-estimated number of evaluations in the earlier version is revised.
- Emphasis is put on complicated TSS problems, with up to 308 design parameters. Some of the test problems are proposed in this study, by converting a simpler problem to a complicated TSS problem.

In the next section, previous studies on two main components of FSD-ES are briefly reviewed. The improved method, called FSD-ES-II, is explained in Section 3. A test suite consisting of complicated TSS truss optimization problems is formed in Section 4 and results from FSD-ES-II are compared to the best available results in the literature in Section 5.

2. Main components

FSD-ES utilized two underlying concepts: The ES part performs the stochastic global search in the whole search space while the FSD part optimize the size variables of the solution provided by the ES. By using FSD, problem specific knowledge is incorporated to the algorithm. These two parts are briefly discussed in this section.

2.1. Fully Stressed Design (FSD)

In general, the truss optimization problem can be formulated as follows:

$$\begin{aligned}
&\text{Minimize } weight(\boldsymbol{\theta}) = \rho\sum_{i=1}^{N_m} A_i L_i \\
&\text{subject to} \\
&u_{kl} \leqslant u^{\text{all}}, \quad k = 1,2,\ldots,DN_n,\ l = 1,2,\ldots,N_l, \\
&|\sigma_{il}| \leqslant \sigma^{\text{all}}, \quad i = 1,2,\ldots,N_m,\ l = 1,2,\ldots,N_l, \\
&A_i \in \mathbb{A}, \quad i = 1,2,\ldots,N_m,
\end{aligned} \tag{1}$$

where $\boldsymbol{\theta}$ determines a design. N_m, N_n and N_l are the number of members, nodes and load cases respectively. $D = 2$ for planar and $D = 3$ for spatial trusses. σ_{il} is the stress in the i-th member and u_{kl} is the displacement of the k-th degree of the truss under the l-th load case, respectively. u^{all} and σ^{all} denotes the allowable limit for node displacement and member stress, respectively, which are known or can be computed. A_i and L_i are the cross-section area and the length of the i-th member, respectively. ρ is the density of the truss material and $\mathbb{A}$ is the given set of available sections.

FSD is based on iterative resizing of the member cross section areas to minimize the truss weight such that all constraints are satisfied. No change on the topology or shape is made and thus, the only design parameters are member sections, $\boldsymbol{A}$. The force distribution is assumed to be independent of the member cross section areas. For this case, the effect of each member on displacement can be computed using the unit load method:

$$u_{kl}(\boldsymbol{A}) = \left|\sum_{i=1}^{m} \frac{c_{ikl}}{A_i}\right|, \quad c_{ikl} = \frac{f_{ik} F_{il} L_i}{E} \tag{2}$$

In Eq. (2), f_{ik} is the axial force in the i-th member when a unit load is applied to the k-th degree of freedom of the truss. F_{il} is the axial force in the i-th member under the l-th load case and E denotes the modulus of elasticity of the truss material. According to Eq. (2), each displacement constraint depends on many or even all sections, therefore, solving the resizing problem, in general, is not easy. In a study [3], a two-step approach was employed such that in the first step, member sections are increased or decreased so that all stress constraints are satisfied and activated. In the second step, satisfaction of displacement constraints is pursued, while, no reduction in the cross section areas is allowed.