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Non-physical finite element method: Multiple material discontinuities

R. Darvizeh, K. Davey *

School of Mechanical, Aerospace and Civil Engineering, The University of Manchester, UK

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ABSTRACT

The theory proposed in the paper provides a new approach for the modelling of multiple material discontinuities. The scope of this work is restricted to numerical methods and in particular an approach that utilises the benefits of traditional continuous finite element approximations but enhanced with an increased capacity for handling material discontinuities. The approach is founded on transport forms of the governing conservation laws describing discontinuous physics by means of bespoke moving control volumes and the non-physical field concept. The theory is demonstrated through its application to steady-state and transient flyer impact-plate problems for which analytical results are available.

1. Introduction

The concept of the non-physical method for the modelling of multiple material discontinuities in solidification was introduced in reference [1] for dealing with a strong discontinuity in enthalpy and a weak discontinuity in temperature. The method proposed here is a further investigation for modelling of the general discontinuous problems rather than being limited to solidification phenomena [1,2]. The work also extends previous work [3] limited to the modelling of general single material discontinuities using the non-physical finite element method (NPFEM). The founding idea underpinning the NPFEM is the ability to define non-physical variables via moving control-volume transport equations along with their representation using traditional approximations common to the finite element method (FEM). A feature of the non-physical approach is that it provides an exact description of the underpinning physics describable by transport equations. Any discontinuous behaviour in a physical field variable is represented exactly by a continuous non-physical field on which a non-physical source is superimposed at a discontinuity. Expressing the governing equations in the integral transport equation form facilitates the use of multiple control volumes. To describe discontinuities in the NPFEM involves enclosing each discontinuity in a moving control volume (CV), which are themselves moving in another moving CV. Collapsing a moving CV at a discontinuity reveals the source-like behaviour in the non-physical field. This limiting process provides a strong description of the behaviour at a discontinuity and new insight into this aspect is provided in this paper.

The definition of non-physical variables via transport equations is ideal for the precise isolation of material discontinuities and hence their description. The ability to focus and collapse a control volume at any point of a discontinuity means that complex geometrical branched discontinuities can in principle be represented. A complex discontinuous physical field is describable by a continuous non-physical field with a source-like behaviour superimposed at discontinuities. A Galerkin weighted-transport method is applied to arrive at a weak form of the weighted transport equations (TEs). Due to the fact that non-physical fields are essentially continuous (with limiting continuity at a physical discontinuity) traditional continuous Galerkin shape functions can be employed. The NPFEM replaces any governing conservation equation for a physical field ψ, by an equivalent governing equations written in terms of its associated non-physical counterpart $\widehat{\psi}$. This substitution facilitates the solving of discontinuous problem in the framework of the traditional Galerkin finite element method (GFEM). Shown in Fig. 1 is a 1-D schematic of a discretisation of a non-physical variable using continuous shape functions. Here ψ is the discontinuous physical field, $\widehat{\psi}$ is the non-physical variable, whilst $\widehat{\psi}'$ is the non-physical source and $\widehat{\psi}_h$ represents the approximation of $\widehat{\psi}$ using linear shape functions.

On first inspection the NPFEM may appear similar to the extended finite element method (XFEM) and the discontinuous Galerkin finite element method (DGFEM) which are the two main FE-based shock capturing techniques. However, the XFEM for example enriches the classical Galerkin shape functions and consequently extends the Sobolev space of the traditional GFEM [4]. This extension can result in singularity and instability particularly for high-rate discontinuous phenomena [5,6]. In addition, the DGFEM

* Corresponding author. Tel.: +44 (0)161 306 3834.
E-mail address: keith.davey@manchester.ac.uk (K. Davey).

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violates the space of the classical GFEM and works on a space termed the broken (semi) Sobolev space [7]. It permits the introduction of discontinuities at the element edges and thus provides a weak continuity between field variables, which can be problematic for solution stability [8,9]. The singularity and instability issues that arise in XFEM and DGFEM can intensify for high-rate multiple discontinuities. In contrast to the XFEM and DGFEM, the NPFEM relies completely on the framework of the traditional GFEM and consequently can readily cater for high-rate multiple material discontinuities. The full details of the review and theoretical comparison between NPFEM, XFEM and DGFEM can be found in reference [3].

Another recently developed competitor to the XFEM is the numerical manifold method (NMM) [10,11]. The NMM is a promising method for modelling material and geometrical discontinuities and can be categorised as a continuum–discontinuum numerical technique [12,13]. The NMM employs a dual cover system (mathematical and physical) with the mathematical cover independent of any discontinuities present. This feature enables the NMM to model any kind of weak/strong discontinuities without recourse to adaptive or re-meshing [11]. The physical cover is obtained by intersecting the mathematical cover with physical boundaries and discontinuities [12,14]. Degrees of freedom are attached to the physical cover and consequently the global approximation of a field is not based on nodal-weighted averages [14,15]. This aspect can result in rank dependency/deficiency issues in the case of local linear approximations [13,15]. A particular concern with the NMM is the construction of elements on a physical cover, which can involve irregular and complex geometries [12]. Moreover, the method requires an extra cover refinement algorithm near a crack tip [11] along with special treatment for the jump conditions at material interfaces [19]. Some of these issues have been addressed and improved upon in the recently published literature [10,11,14,15]. The approach, when contrasted with the XFEM, is reported to be a more robust discontinuity-capturing, numerical technique. The XFEM is known to suffer from issues with ill-conditioning of the cut-element stiffness matrices [14] and due to its cover-based property the NMM can naturally handle multiple, branched and intersected discontinuities, where usually the XFEM fails [16].

In contrast with the NMM, the proposed NPFEM does not involve dual meshes and is founded firmly on the classical FEM, a feat achieved by ensuring that only continuous fields are approximated. This in effect increases the capability of classical FE-based numerical algorithms for modelling material discontinuities and has the potential to be directly incorporated into commercial software. It also has the potential for incorporation into a mesh-free formulation although this consideration is not the focus of the present work. The single discontinuity version of the NPFEM has been tested for the high-rate crushing of cellular materials and the results are presented in reference [17].

In this paper, a formulation of the NPFEM is presented which permits the removal of the non-physical source $\widehat{\psi}'$ from the governing weighted transport equations yet sufficiently represent the underlying physics at a discontinuity. The NPFEM for multiple material discontinuities is tested on steady-state and transient case studies and excellent results are obtained.

The outline of the paper is as follows. In Section 2 the preliminary concepts of the integral form of transport conservation equations in an arbitrary Lagrangian Eulerian framework are given. The non-physical variables and their equivalent conservation governing equations are also introduced. The behaviour of the non-physical field at the place of discontinuity is presented along with the non-physical equivalent equations, which are developed for multiple discontinuities. The annihilation of multiple discontinuities from the NPFEM formulations is proved mathematically in Section 2.4. In Section 3, the finalised form of the FE formulation for the non-physical equivalent transport equations is presented. These equations are shown to be absent of the multiple discontinuities that are present in the original formulation. In Section 4, the robustness and performance of the NPFEM is illustrated though three cases studies. And finally, Section 5 concludes with important concluding remarks.

2. Theoretical background

In this paper, integral forms of the transport equations are employed to represent the governing conservation laws. The use of integral forms in comparison with the differential equations permits the direct incorporation of discontinuous physical fields. This feature arises from the fact that integration increases the degree of smoothness and can cope with discontinuous functions whilst classical differentiation cannot. Therefore, integral transport equations are the more generic form of representation of the conservation equations since they can be reduced to the differential form, should this exist.

2.1. Weighted conservation transport equations

The formulation proposed here can in some sense be considered as a generalised form of the arbitrary Lagrangian–Eulerian (ALE) formulation. This formulation permits independent control volume (CV) movement in a computational reference system (CRS). The Material reference system (MRS), spatial and CRS co-ordinates are denoted by $\boldsymbol{X}$, $\boldsymbol{x}$, and $\boldsymbol{\chi}$, respectively. An important velocity is $\underline{v}^* = D^*\boldsymbol{x}/D^*t = \partial\boldsymbol{x}(\boldsymbol{\chi},t)/\partial t$, which serves to transport the computational control volume to which any mesh is attached. Its formal definition involves a reference coordinate system (fixed CV) with coordinate $\boldsymbol{\chi}$. Apart from this definition the reference system is to play little part in this paper as all analysis is preformed on the moving CV. The definition of the derivative D^*/D^*t is similar in form to that for the well-established material derivative $\underline{v} = Dx/Dt = \partial x(\boldsymbol{X},t)/\partial t$, where $\boldsymbol{X}$ belongs to the material reference system. The use of the derivative D^*/D^*t, when applied to an integral of the form $\int_\Omega \rho\psi dV$, is intended to immediately relay that Ω is a control volume transported with velocity $\underline{v}^*$.

One of the deficiencies of $\underline{v}^*$ is that it does not generally match the normal velocity of any discontinuity passing through the domain Ω. This failing necessitates the definition of an additional velocity $\underline{v}^+$, which by design does so but in addition matches the normal velocity of $\underline{v}^*$ on Γ the boundary for Ω. A conservation law for a moving domain Ω defined by the diffeomorphism $\boldsymbol{x}(\boldsymbol{\chi},t)$ is

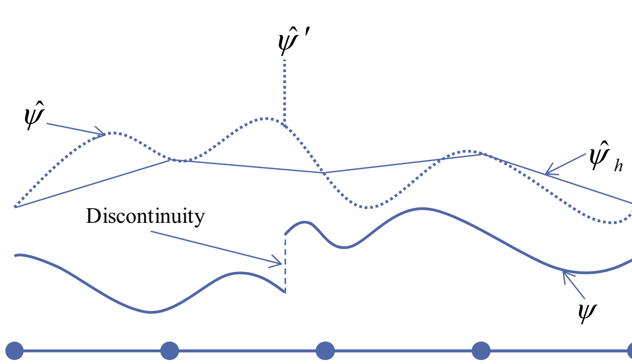

Fig. 1. Schematic 1-D modelling of a discontinuous function in the NPFEM.