Computers and Structures 159 (2015) 41–60

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

On the use of stochastic spectral methods in deep excavation inverse problems

Antonio Cañavate-Grimal [a,*], Antonio Falcó [a], Pedro Calderón [b], Ignacio Payá-Zaforteza [b]

[a] ESET, Universidad CEU Cardenal Herrera, Spain
[b] ICITECH, Universitat Politècnica de València, Spain

ARTICLE INFO

Article history:
Received 6 November 2014
Accepted 10 June 2015

Keywords:
Bayesian inference
Inverse problems
Spectral methods
Stochastic finite elements
Deep excavations

ABSTRACT

The back analysis or inverse analysis of the field instrumentation data is a common technique to ascertain the design parameter validity in deep excavation projects. That analysis is a process full of uncertainties and relies greatly on the expert judgement. Furthermore, deep excavation geotechnical models tend to be computationally very expensive making the inverse analysis a very lengthy process. In this paper, a Bayesian-type methodology to solve inverse problems which relies on the reduction of the numerical cost of the forward simulation through stochastic spectral surrogate models is presented. The proposed methodology is validated with three calibration examples.

1. Introduction

Soil is a highly non-linear material whose strength and stiffness depends on stress and strain levels. Numerous constitutive models have been developed to simulate the most important features of soil behaviour [1–3] although there is no agreement on which is the best to model a particular type of soil. The choice of the model rests on the available soil information and the particular design situation. In addition, determination of soil parameters is a difficult task due mainly to the ground heterogeneity, the boundary conditions uncertainty (water table position, layering, ...), the disturbance suffered by soil specimens during geotechnical sampling and the small quantity of soil surveyed compared to the mass of ground affected by any foundation. Hence, the bulk information yielded by the field surveying and laboratory testing program must be interpreted by an experienced engineer, adding more uncertainty to the choice of the constitutive model and its parameters [4]. In order to ascertain the parameter validity, it is common in geotechnical engineering to perform back analysis or inverse analysis procedures from field instrumentation. The field observations might not be as precise as desired due to the hard site conditions. Hence, to solve a geotechnical inverse problem means to estimate partially known parameters from indirect noisy observations. This is not an academic issue since it has practical applications, for instance, the information recorded during the early stages of the construction might be used to update and validate the initial design predictions. The back analysis is a tool which enables to gain insight and to understand better the soil-structure system behaviour [5].

Inverse problem resolution is not new since numerous authors have studied it previously [6–16]. Many of those studies [6,8–11,14] address the back analysis as an optimization problem (i.e. obtaining the set of parameters which minimize an objective function). That approach leads to estimated parameters which reliability is generally unknown [15]. As the model might not be able to reproduce perfectly the actual response and the observations might suffer measurement errors, the solution should take into account the model and observations uncertainties. For that reason, the Bayesian approach is the appropriate methodology to solve inverse problems [17,18]. In geotechnical engineering, the solution of statistical inverse problems can be a computationally intensive task. The numerical burden arises mainly in two ways [19]: (i) the large number of parameters that the model might require and (ii) the computational cost that might require a single realization to run.

The main objective of this paper is to present a Bayesian methodology to determine at low numerical expense the model parameters from the observed response at one construction stage of a deep excavation. To that end, the Bayesian methodology is briefly outlined in the first part of this paper. The Bayesian methods regard the model parameters as random variables which are updated once a set of observations is known. As the solution of the problem is posed in terms of random variables, the stochastic

* Corresponding author at: ESET, Universidad CEU Cardenal Herrera, C/ San Bartolomé 55, Alfara del Patriarca, 46015 Valencia, Spain.
E-mail address: antonio.canavate@uch.ceu.es (A. Cañavate-Grimal).

http://dx.doi.org/10.1016/j.compstruc.2015.06.009
0045-7949/

spectral representation can be used to reduce the numerical burden [20,21]. The random variable spectral representation is the backbone of the non-intrusive stochastic finite element methods (SFEM) [22,23] and hence those methods are briefly described. Once the theoretical framework has been established, the proposed methodology to solve inverse problems is adequately presented. The spectral approach raises two benefits (i) the substantial reduction of computational cost when performing the optimization calculations on the surrogate model constructed by means of SFEM and (ii) the possibility of an analytical computation of the statistical relationship between the different observations (i.e. the covariance matrix). However, the surrogation brings a modelling error affecting the solution. One of the main findings of this paper is the algorithm developed to estimate the surrogation error at low numerical expense. Finally, the proposed methodology is validated by the study of three calibration problems of increasing complexity.

2. Inverse problem solution

The connection between the parameters $\mathbf{m}$ and the observations $\mathbf{d}$ defines the following deterministic forward or response model $\mathbf{g}(\mathbf{m})$, namely:

$$\mathbf{m} \rightarrow \mathbf{d} = \mathbf{g}(\mathbf{m}). \tag{1}$$

The predicted values cannot be identical in general to the observed values due to observational and modelling errors. The goal of the inverse problem is to infer the N model parameters $\mathbf{m}$ from a set of n imperfect observations $\mathbf{d}$. In the Bayesian approach, $\mathbf{m}$ and $\mathbf{d}$ are vectors of random variables. $\rho_d(\mathbf{d})$ is the probability density function (pdf) of the observations and $\rho_m(\mathbf{m})$ encodes any available prior knowledge about the inputs. The key point of Bayesian inference is the way that the *a priori* distribution $\rho_m(\mathbf{m})$ is updated to the *a posteriori* distribution $\sigma_m(\mathbf{m})$ once a set of observations $\mathbf{d}_{obs}$ is known.

The difference between the observed values $\mathbf{d}_{obs}$ and the actual response $\mathbf{d}$ is the observational error. The vector of n residuals $\boldsymbol{\varepsilon}_D$ is regarded in this paper Gaussian of zero mean:

$$\boldsymbol{\varepsilon}_D = \mathbf{d} - \mathbf{d}_{\text{obs}} \sim \mathrm{N}(\mathbf{0}, \mathbf{C}_D), \tag{2}$$

where the covariance matrix $\mathbf{C}_D$ measures the size of those residuals and gives the dependence between them. The *a priori* observation distribution can be written as [24]:

$$\rho_d(\mathbf{d}) \propto \exp\left[-\frac{1}{2}(\mathbf{d} - \mathbf{d}_{\text{obs}})^T \mathbf{C}_D^{-1}(\mathbf{d} - \mathbf{d}_{\text{obs}})\right], \tag{3}$$

As $\mathbf{m}$ and $\mathbf{d}$ are regarded independent random variables, the joint pdf is given by (see Fig. 1a):

$$\rho(\mathbf{d}, \mathbf{m}) = \rho_d(\mathbf{d})\rho_m(\mathbf{m}) \tag{4}$$

If the forward model $\mathbf{g}(\mathbf{m})$ were perfect (i.e. modelling errors free), each parameter vector would yield only one observation vector. Nevertheless, most often, the underlying physical theory lacks of some fundamental knowledge or fails to achieve a perfect parametrization [17]. Therefore, the joint probability density $\Theta(\mathbf{d}, \mathbf{m})$ is required to describe the correlations that correspond to the physical theory, together with the inherent uncertainties of the theory (see Fig. 1b).

The modelling residual vector is the difference between the model $\mathbf{g}_{real}$ (perfect but unknown) and the available forward model $\mathbf{g}(\mathbf{m})$ (known but imperfect). Again, that residual is assumed in this paper as an additive Gaussian of zero mean:

$$\begin{aligned} &\boldsymbol{\varepsilon}_G(\mathbf{m}) = \mathbf{g}_{real} - \mathbf{g}(\mathbf{m}) \\ &\boldsymbol{\varepsilon}_G \sim \mathrm{N}(\mathbf{0}, \mathbf{C}_G) \end{aligned} \tag{5}$$

where $\mathbf{C}_G$ is the model covariance matrix giving information about the size of the residuals and the correlation between them. The matrix $\mathbf{C}_G$ could be regarded as the model epistemic error and expert judgement is required to estimate it. When the dependence of $\mathbf{d}$ on $\mathbf{m}$ is mildly non-linear, $\Theta(\mathbf{d}, \mathbf{m})$ can be expressed as [17]:

$$\Theta(\mathbf{d}, \mathbf{m}) \propto \exp\left[-\frac{1}{2}(\mathbf{g}(\mathbf{m}) - \mathbf{d})^T \mathbf{C}_G^{-1}(\mathbf{g}(\mathbf{m}) - \mathbf{d})\right]. \tag{6}$$

The conjunction of the information contained in $\rho(\mathbf{d}, \mathbf{m})$ and $\Theta(\mathbf{d}, \mathbf{m})$ results in the updated or posterior probability density function. The solution of the inverse problem is a new or *a posteriori* pdf $\sigma_m(\mathbf{m})$ which incorporates the information given by the observed values $\mathbf{d}_{obs}$ and it is consistent with the amount of modelling and observational uncertainty [17] (see Fig. 1c):

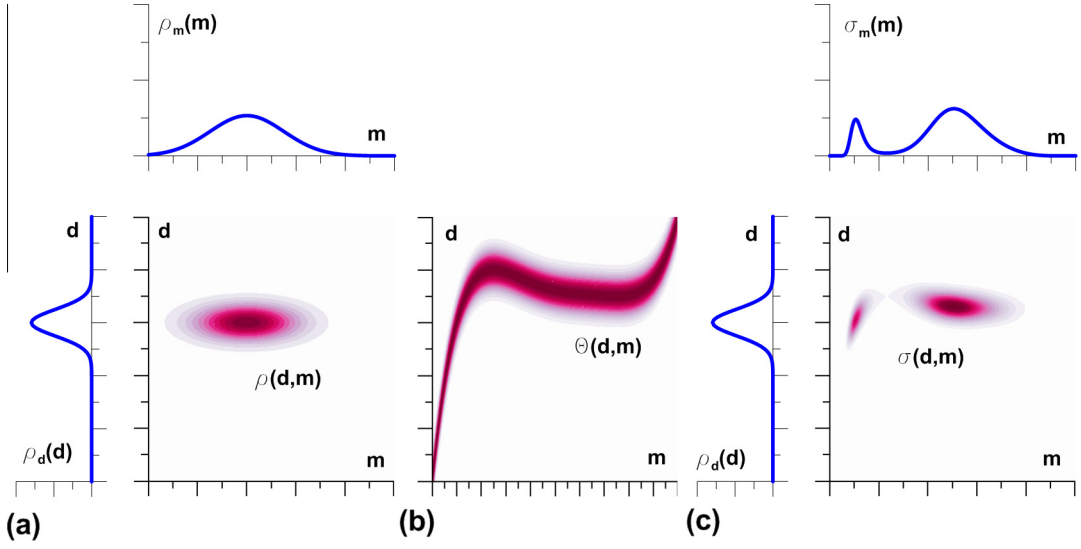

Fig. 1. Inverse problem resolution (a) *a priori* parameter and observation marginals, $\rho_m(\mathbf{m})$ and $\rho_d(\mathbf{d})$ respectively, and joint probability functions, $\rho(\mathbf{d}, \mathbf{m})$, (b) model joint pdf, $\Theta(\mathbf{d}, \mathbf{m})$, (c) *a posteriori* parameter and observation joint pdf, $\sigma(\mathbf{d}, \mathbf{m})$, and the *a posteriori* marginal parameter probability distribution function (pdf), $\sigma_m(\mathbf{m})$ (adapted from [17]).