Computers and Structures 158 (2015) 42–60

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Numerical analysis of the mechanical role of the ribs in groin vaults

G. Lengyel*, K. Bagi

Budapest University of Technology and Economics, Department of Structural Mechanics, 1111 Budapest, Mûegyetem rkp. 3, Hungary

ARTICLE INFO

Article history:
Received 15 April 2015
Accepted 26 May 2015

Keywords:
DEM
Masonry cross vault
Masonry

ABSTRACT

The role of ribs in the mechanical behavior of masonry cross vaults has been the subject of intense debates since the 19th century. Literature on the subject diverges from considering the ribs as the main load-bearing units which carry the weight of the masonry web, to the opinion that the ribs are merely decorations. This research focused on the simplest type of cross vaults, i.e. groined vaults formed by the intersection of two semicircular cylindrical mid-surfaces. Instead of the widely used Limit State Analysis which is reliable only if specific conditions are satisfied, discrete element modelling (the commercial code 3DEC, based on an explicit time integration scheme), and a classical finite element code (ANSYS) was applied in the investigations. In the applied DEM code (3DEC) the elements (corresponding to the voussoirs) may slide along each other, and can be separated from their neighbors in any directions; and new contacts may be formed between them, in a computationally efficient automatized manner.

1. Introduction

The application of ribbed vaults appeared in Europe already in the 10th century, presumably for Muslim or Armenian inspiration [1]. The idea became widespread in the late Romanesque era, and then became a fundamental feature of Gothic structures. In Christian architecture the ribs are rather thick and strong in comparison to the thin, dense, decorative networks of the seemingly fragile ribs of Arabic vaults; consequently, their mechanical function is also different. In the present paper we shall focus on the former ones, specifically on the earliest version of ribbed vaults which became widely applied throughout Europe, i.e. late Romanesque groin vaults. The interested reader can find a detailed overview and discussion on the hypothesis about the possible origins of ribbed vaulting, with an emphasis on the Islamic version, in [1].

Early Romanesque cross vaults consisted of two semicircular barrel vaults, usually intersecting at a right angle. To build such a vault, a complex system of scaffolding and centering had to be erected in order to define the shape of the intrados and lay the masonry. For larger spans this made the construction process rather complicated and inconvenient. In addition, in such structures the intersection lines formed by the two surfaces (i.e. the diagonal groins) were rather weak and attracted damages.

An efficient solution came into general use in Europe from approximately the 12th century. Along the planned intersection lines ashlar ribs were erected first, which then played the role of permanent centering, during construction as well as through the lifetime of the structure. The masonry shell was divided into smaller domains this way, which made the construction process easier and larger spans could consequently be overcome; in addition, the ribs also had an aesthetic effect. This solution was proven definitely successful and quickly spread about, then became a main characteristic of Gothic architecture in the following centuries.

Regarding Gothic ribbed vaults, a fundamental question has been under debate since the 19th century: are the ribs the main load-carrying members and the masonry shell is mostly a passive load on it, or on the contrary, the masonry web carries its own weight and the ribs only provide an additional reinforcement and a visual impression of stability. Huerta [2] gave a thorough overview on that debate, from the early hypothesis (e.g. Willis [3] who made a distinction between "mechanical ribs" sustaining the vault and "decorative ribs" applied mainly for aesthetic functions) till sophisticated numerical investigations (e.g. Barthel [4]). Based on the Safe Theorem of plastic limit state analysis applied to masonry structures (Heyman [5]), Huerta pointed out that the question itself was wrong: since the internal force system in a masonry structure is extremely sensitive to slight changes in the geometrical boundary conditions: small soil settlements or leaning of supporting walls, etc. may abruptly change the stress distribution which was valid under previous circumstances, and it means that either the ribs or the masonry shell, or both of them in a variable proportion, may therefore be the main load-bearing component, depending on the current conditions of the structure,

* Corresponding author.
E-mail addresses: lengyelg@mail.bme.hu (G. Lengyel), kbagi@mail.bme.hu (K. Bagi).

http://dx.doi.org/10.1016/j.compstruc.2015.05.032
0045-7949/

subjected to change continuously. This conclusion is, of course, also valid for Romanesque ribbed vaults.

However, the question of structural functions of the rib, i.e. the differences between the mechanical behavior of unribbed and ribbed vaults, remained an open issue. According to Heyman [5] the ribs resolve those stress concentrations which would otherwise occur around the four corners where the vault is supported from below. Alexander et al. [6] found that the ribs strengthen the vault just along its weakest lines. Other, still unrevealed effects may also be present. The aim of the present study is therefore to provide numerical simulation and comparison of the behavior of unribbed and ribbed cross vaults carrying their self-weight during different displacement histories of their boundaries. The aim is not to find "the current state" of stress for a given geometry and supports: instead, our intention is to survey the set of those possible states which occur for a wide spectrum of disturbed boundary positions, and find out what differences are caused by the existence of the ribs both in the stress states and in the failure modes.

The numerical investigations presented in the paper focus on the simplest and earliest type of ribbed vaults, shown in Fig. 1: two semicircular barrel vaults having equal radius, intersecting above a square plan.

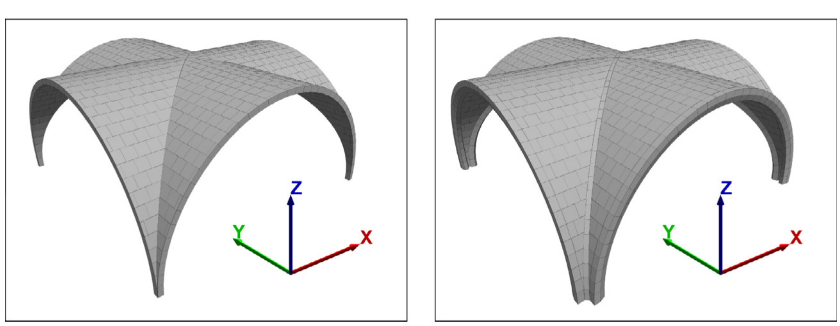
Several different computational tools exist for the analysis of masonry behavior. They can be categorized into three main groups.

- Limit State Analysis methods (e.g. Thrust Network Analysis, Block & Ochsendorf [7]) have a limited validity if they are based on the assumption that frictional sliding does not occur. Those models which allow for the possibility of failure with frictional sliding (e.g., Livesley [8,9]; Orduña & Lourenço, [10]; D'Ayala & Tomasoni, [11]) are definitely more suitable for our purposes, though they are computationally rather expensive.
- Continuum-based techniques like, e.g., the finite element method (FEM) may provide valuable insight into the behavior of the structure, but those versions which are most suitable for the analysis of the failure regime (nonlinear FEM with no-tension constitutive behavior, or application of contact elements reflecting Coulomb-type behavior at pre-defined surfaces) are also rather inefficient from computational point of view. Thus, in our researches FEM was applied only for that fairly limited range of the behavior where linear elasticity could be assumed. The results obtained this way are valid only for tension-resisting states of the structure, and should be accepted with reservations. However, the sophisticated output systems and visualization possibilities offered by recent commercial software packages can significantly contribute to the understanding of the internal state of the analyzed structure; this is why we did not completely exclude FEM from the analysis.
- Discrete element modelling (DEM) considers the structure as a collection of separate blocks each of which is able to move and deform independently of each other. The blocks may come into contact with each other where contact forces are transmitted, causing stresses and deformations inside the blocks. The blocks may also frictionally slide along each other. The contact creation, sliding and separation is automatically followed in a computationally efficient manner in DEM. These characteristics make DEM particularly suitable for masonry analysis; before, during and after failure, and this is why we chose DEM to serve as the main tool of the investigations.

There were several methodological differences between the FEM and DEM models applied in the present paper. The FEM model was based on compiling and solving the global equilibrium equations of a quasi-static system according to the usual displacement method: $\mathbf{Ku} = \mathbf{f}$, where $\mathbf{f}$ was the vector of forces reduced to the nodes of the finite elements, $\mathbf{u}$ was the basic unknown, i.e., the displacement vector that moved the system from the initial unloaded geometry to the final equilibrium state corresponding to the external forces acting on the structure (some of these displacements were prescribed in the case of support displacement analysis); and $\mathbf{K}$ was the global stiffness matrix describing the geometrical and material data of the simulated structure.

The DEM analyses were based on simulating the motions of the individual nodes of the discrete elements in time, with the help of an explicit time integration of Newton's force-acceleration law: $v_{i+1/2} = v_{i-1/2} + \frac{f_i}{m}\Delta t$, where v denotes the velocities, Δt is the length of the finite time step considered, m is the mass assigned to the analyzed node, and f_i is the force resultant reduced to that node (see details in Section 4.1.1). In other words, the discrete elements were subdivided into regions belonging to the different nodes, and their motion was followed in time. The main differences between the applied FEM and DEM technique were the following:

- The FEM solution was time-independent, only small displacements could be analyzed, while DEM was able to produce finite (i.e. large) displacements received from a series of small incremental time steps.
- FEM used a global stiffness matrix, while in the DEM code all individual nodes were considered independently, and the modification of the contact forces between the elements was not considered during a time step (contact forces were upgraded only after receiving the modified position of the nodes).
- The usual continuity conditions between adjacent elements were satisfied in the FEM model, while such conditions were not applied between the discrete elements in the DEM model.



(a) (b)

Fig. 1. Geometry of the simulated cross vaults: (a): Groin vault; (b): Ribbed vault.