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Topology optimization of large-scale structures subjected to stationary random excitation: An efficient optimization procedure integrating pseudo excitation method and mode acceleration method

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ABSTRACT

Structural topology optimization related to dynamic responses under stationary random force excitation is investigated in this paper. It is shown that the commonly used Complete Quadratic Combination method (CQC) in previous optimization work is not only computationally expensive but also results in non-convergent design pattern due to the low computing accuracy of random responses for large-scale problems. To circumvent these difficulties, an efficient and accurate optimization procedure integrating the Pseudo Excitation Method (PEM) and Mode Acceleration Method (MAM) is introduced into the dynamic topology optimization. In this framework, random responses are calculated using the PEM to ascertain a high efficiency over the CQC. More importantly, the accuracy of random responses is improved indirectly by solving the pseudo harmonic responses involved in the PEM with the help of the MAM. Numerical examples fully demonstrate the validity of the developed optimization procedure and its potential applications in practical designs.

1. Introduction

As is well-known, the topological layout of a structure has a dominant impact upon its dynamic performances and responses. Since the work of Bendsøe and Kikuchi [1], topology optimization method has been widely developed and applied in engineering designs especially for static structures. However, many engineering structures experience not only static loads but also random excitations such as earthquake ground motions, ocean wave induced forces, aerodynamic and turbulent pressures and so on. In general, random excitations can be classified into stationary and non-stationary random excitations. If the ensemble averages for a random excitation were time-independent, the excitation would be stationary. Otherwise, it would be non-stationary. To a large extent, some secondary structure designs were mostly based on stationary random excitations [2] that were greatly considered in the advanced formulations of dynamic optimization problems [3–6].

However, results about dynamic topology optimization are few up to now due to the inherent complexity of the problem. Representative works were mostly limited to problems with a small number of degrees of freedom (DOFs). For example, Rong et al. [7,8] optimized the structural topology using the ESO method with stationary random responses constrained in design. Zhang et al. [9] dealt with topology optimization of multi-component structures under both static loads and stationary random excitations using density method. In fact, all the above works were carried out by means of the CQC. As the CQC was cost-ineffective in random analysis, the PEM [10–14] was thus introduced to transfer the solving of random responses into the solving of pseudo harmonic responses. Although both methods can completely achieve the same solution with the same number of structural modes, the efficiency of the PEM is much higher than the CQC. With this advantage, Lin et al. [15] adopted the PEM as an efficient optimization procedure in the maximization of the energy harvesting performance under stationary random excitation. Nevertheless, how to deal with dynamic topology optimizations of large-scale problems still rests a great challenge.

Another fatal problem in dynamic topology optimization of large-scale problems is the severe convergence difficulty. In authors' previous work [16], it was revealed that the convergence would be very poor even in design optimization of the harmonic response if the latter was not accurate enough. Similarly, as observed in Section 3.3, the truncation modes in both the CQC and the PEM would also introduce computing errors into the

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random responses and even lead to unsatisfactory structure configurations after topology optimization. Theoretically, the errors could be reduced by adopting a great number of modes in the analysis, but the computing would be prohibitive especially for large-scale problems. Meanwhile, how to select a proper number of modes in advance is also challenging. Therefore, adopting a great number of modes is neither a reasonable nor practical strategy. In fact, the accuracy of random response can indirectly be improved by increasing the accuracy of pseudo harmonic responses computed within the framework of the PEM. Shi et al. [17] introduced the MAM to replace the MDM involved in the conventional PEM for structural analysis under multi-support excitations. This combination of MAM and PEM provided a great superiority in accuracy, computing efficiency and deserves further explorations in dynamic optimization of large-scale problems. With this motivation, the combined method of PEM and MAM is further extended and integrated into the optimization procedure to deal with large-scale dynamic optimization problems under stationary random excitation. In this paper, sensitivity analysis involved in this procedure is also presented. Based on numerical examples, comparative studies are made with existing methods to highlight the effectiveness of the developed optimization procedure.

2. Analysis methods of structural dynamic responses under stationary random force excitation

2.1. Complete quadratic combination method (CQC)

Now, consider a discretized n-DOF structure subjected to stationary random force excitation. The motion equation can be written as

$$\boldsymbol{m}\ddot{\boldsymbol{u}}(t)+\boldsymbol{c}\dot{\boldsymbol{u}}(t)+\boldsymbol{k}\boldsymbol{u}(t)=\boldsymbol{b}\boldsymbol{p}(t) \tag{1}$$

where $\boldsymbol{m}$, $\boldsymbol{c}$, $\boldsymbol{k}$ represent the mass matrix, damping matrix and stiffness matrix. $\boldsymbol{p}(t)$ is a d-dimension stationary random force vector of non-zero values, whose power spectral density (PSD) matrix is of d-dimension and denoted by $\boldsymbol{S}_p(\omega)$. Notice that $\boldsymbol{b}$ is a $n\times d$ transformation matrix representing the force distribution. In this paper, $\boldsymbol{bp}(t)$ is assumed to be white-noise excitation with zero mean value, i.e., a uniform power spectral density over the frequency interval. By introducing the notation

$$\boldsymbol{u}(t)=\boldsymbol{\varphi}\boldsymbol{z}(t)=\sum_{i=1}^{n}\boldsymbol{\varphi}_i z_i \tag{2}$$

where $\boldsymbol{z}(t)$ is the vector of generalized coordinates. Under the assumption of classical damping with ζ_i being the ith damping ratio, following relations then hold

$$\begin{cases}\boldsymbol{\varphi}^T\boldsymbol{m}\boldsymbol{\varphi}=\boldsymbol{I}\\ \boldsymbol{\varphi}^T\boldsymbol{k}\boldsymbol{\varphi}=diag(\omega_i^2)\\ \boldsymbol{\varphi}^T\boldsymbol{c}\boldsymbol{\varphi}=diag(2\zeta_i\omega_i)\end{cases} \tag{3}$$

where ω_i and $\boldsymbol{\varphi}_i$ represent the ith circular eigenfrequency and the ith eigenvector, respectively. The mode shape matrix $\boldsymbol{\varphi}=[\boldsymbol{\varphi}_1\ldots\boldsymbol{\varphi}_n]$ is normalized by mass matrix. Then a number of n uncoupled equations of motion can be obtained by substituting Eq. (2) into Eq. (1) and premultiplying $\boldsymbol{\varphi}^T$.

$$\ddot{z}_i(t)+2\zeta_i\omega_i\dot{z}_i(t)+\omega_i^2 z_i(t)=\boldsymbol{\varphi}_i^T\boldsymbol{b}\boldsymbol{p}(t) \tag{4}$$

By means of the Duhamel integral, the time-domain solution of Eq. (4) is [18]

$$z_i(t)=\int_{-\infty}^{\infty}\boldsymbol{\varphi}_i^T\boldsymbol{b}\boldsymbol{p}(t-\tau)h_i(\tau)d\tau \tag{5}$$

where $h_i(\tau)$ is the unit impulse response function related to the single DOF system of Eq. (4)

$$h_i(\tau)=\begin{cases}\frac{e^{-\zeta_i\omega_i\tau}}{m_i\sqrt{1-\zeta_i^2}\omega_i}\sin\sqrt{1-\zeta_i^2}\omega_i\tau & \tau\geqslant 0\\ 0 & \tau<0\end{cases} \tag{6}$$

with

$$m_i=\boldsymbol{\varphi}_i^T\boldsymbol{m}\boldsymbol{\varphi}_i \tag{7}$$

It follows that

$$\boldsymbol{u}(t)=\sum_{i=1}^{n}\boldsymbol{\varphi}_i z_i=\sum_{i=1}^{n}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^T\boldsymbol{b}\int_{-\infty}^{\infty}\boldsymbol{p}(t-\tau)h_i(\tau)d\tau \tag{8}$$

The autocorrelation function of displacement response $\boldsymbol{u}(t)$ reads [18]

$$\begin{aligned}\boldsymbol{R}_u(\Delta)&=E[\boldsymbol{u}(t)\boldsymbol{u}(t+\Delta)^T]\\ &=\sum_{i=1}^{n}\sum_{k=1}^{n}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^T\boldsymbol{b}\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\boldsymbol{R}_p(\Delta+\tau_1-\tau_2)h_i(\tau_1)h_k(\tau_2)d\tau_1 d\tau_2\right)\boldsymbol{b}^T\boldsymbol{\varphi}_k\boldsymbol{\varphi}_k^T\end{aligned} \tag{9}$$

The PSD matrix of random displacement response $\boldsymbol{S}_u(\omega)$ can then be obtained by Fourier transformation of the above autocorrelation function

$$\boldsymbol{S}_u(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\boldsymbol{R}_{\boldsymbol{u}}(\Delta)e^{-j\omega\Delta}d\Delta=\sum_{i=1}^{n}\sum_{k=1}^{n}H_i^*H_k\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^T\boldsymbol{b}\boldsymbol{S}_p(\omega)\boldsymbol{b}^T\boldsymbol{\varphi}_k\boldsymbol{\varphi}_k^T \tag{10}$$

H_i denotes the frequency domain transfer function between loading and response

$$H_i=(\omega_i^2-\omega^2+2j\zeta_i\omega_i\omega)^{-1} \tag{11}$$

with $j^2=-1$. Actually, it is almost impossible to use all the n modes in the computing process of Eq. (10) especially for large-scale problems. Suppose l is the number of modes employed in the computing with $l\ll n$, Eq. (10) is then approximated as

$$\boldsymbol{S}_u(\omega)=\sum_{i=1}^{l}\sum_{k=1}^{l}H_i^*H_k\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^T\boldsymbol{b}\boldsymbol{S}_p(\omega)\boldsymbol{b}^T\boldsymbol{\varphi}_k\boldsymbol{\varphi}_k^T \tag{12}$$

The CQC method [10,11,18] consists in obtaining the PSD matrix of random displacement response by computing Eq. (12) directly. Since the latter involves the cross-correlation terms between all l participant modes, the computing would be very expensive for large values of l.

2.2. Conventional pseudo excitation method (PEM)

Since the PSD matrix $\boldsymbol{S}_p(\omega)$ is Hermitian, it can be decomposed into [11]

$$\boldsymbol{S}_p(\omega)=\sum_{q=1}^{Q}(\boldsymbol{\gamma}_q)^*(\boldsymbol{\gamma}_q)^T \tag{13}$$

in which Q is the rank of $\boldsymbol{S}_p(\omega)$. Therefore, Eq. (12) can be rewritten as

$$\boldsymbol{S}_u(\omega)=\sum_{q=1}^{Q}\left(\sum_{i=1}^{l}\boldsymbol{\varphi}_i\boldsymbol{\varphi}_i^TH_i\boldsymbol{b}\boldsymbol{\gamma}_q\right)^*\left(\sum_{k=1}^{l}\boldsymbol{\varphi}_k\boldsymbol{\varphi}_k^TH_k\boldsymbol{b}\boldsymbol{\gamma}_q\right)^T \tag{14}$$

Suppose

$$\boldsymbol{g}_q(t)=\sum_{i=1}^{l}\boldsymbol{\varphi}_iH_i\boldsymbol{\varphi}_i^T\boldsymbol{b}\boldsymbol{\gamma}_q e^{j\omega t} \tag{15}$$