



On the parallel implementation of a hybrid-mixed stress formulation



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ABSTRACT

This paper reports the parallel implementation of a hybrid-mixed stress finite element model for the analysis of three-dimensional structures. The stress and the displacement fields in the domain and the displacement fields on the static boundary are independently approximated. All field equations are imposed in a weighted residual form and Legendre polynomials are used to define the approximation bases. The parallel implementation of the model uses a message passing scheme based on the MPI standard. For the parallel solution of the governing system, two distinct approaches are tested. The first corresponds to the use of a third-party parallel direct solver code, named MUMPS. The second uses an in-house developed hybrid solver inspired by the FETI method and designed to deal with the typical pattern of the hybrid-mixed stress model governing system. Some numerical examples are discussed to illustrate the effectiveness and performance of the adopted parallel approach.

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1. Introduction

In recent years, some research effort has been devoted to the development of effective numerical models for the solution of Solid Mechanics problems using non-conventional hybrid and mixed finite element models. These formulations can be classified into three groups, namely hybrid-mixed, hybrid and hybrid-Trefftz, and two models can be distinguished in each formulation [1,2]. The alternative stress and displacement models are designed to produce, under certain conditions, statically and kinematically admissible solutions, respectively. What distinguishes the alternative formulations are the local conditions that the domain approximation bases are constrained to satisfy *a priori*. The approximation bases of the hybrid-Trefftz formulation must locally solve all the fundamental domain conditions. The hybrid formulation is obtained by relaxing this constraint to a subset of the fundamental conditions of the problem, typically either the domain equilibrium or the compatibility conditions. The hybrid-mixed formulation results from accepting approximation bases that locally satisfy none of the domain conditions of the problem under analysis.

In the hybrid-mixed stress model used in this paper, both the stress and displacement fields are simultaneously and independently approximated in the domain of each element [2–4]. The displacements on the static boundary, which is considered to include the inter-element boundaries, are also independently modeled. All

field equations are imposed in a weighted residual form so designed as to ensure that the discrete numerical model embodies all the relevant properties of the continuum field it represents, static kinematic duality and elastic reciprocity.

As none of the fundamental relations in the domain is locally imposed *a priori*, the hybrid-mixed stress formulation enables the use of a wide range of functions to define the approximation bases. This fact makes possible the adoption of sets of functions presenting special properties that make their use very attractive in the context of hybrid-mixed stress computations. Very successful and competitive implementations for the elastic analysis of plate stretching and plate bending problems have been achieved using complete sets of orthonormal Legendre polynomials [5,6], Daubechies wavelet systems [4], systems of wavelets defined on the interval [7] and polynomial wavelets [8]. The hybrid-mixed stress model has been generalized to allow for the physically non-linear analysis of structures using plasticity [9] and isotropic damage models [10–12] and considering both static and dynamic loadings [13,14].

As the hybrid-mixed stress models reported above are implemented using naturally hierarchical approximation bases, very effective p -refinement procedures can be considered and the use of coarse meshes of elements large in dimension (macro-elements) can be adopted [2]. As orthonormal functions are used to define the approximation bases, the solving system is highly sparse and it is possible to establish closed form solutions for the integrations involved in the definition of all structural operators in linear analysis [6–8]. Numerical integration schemes can

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be fully avoided, with clear advantages both in terms of accuracy and numerical performance.

To ensure the numerical efficiency of the computations, and due to the large number of degrees of freedom that may be reached and the global governing system pattern which is typical in hybrid-mixed stress models, it is necessary to use adequate techniques to store, manipulate and solve large sparse systems of equations. Both direct and iterative algorithms have been tested and assessed [15] namely those based on LDU factorizations and on conjugate gradients with different preconditioning conditions.

Only sequential processing algorithms have been adopted so far. However, the hybrid-mixed stress models appear to be, by nature, very appropriate for parallel processing, mostly due to the high sparsity of the global governing system and its non-overlapping block structure. The main goal of this paper is to report a parallel implementation of the hybrid-mixed stress formulation. Substantial improvements in terms of numerical performance are expected, specially when three dimensional structures are to be analyzed. As reported in [16–19] the use of hybrid-mixed stress models for the analysis of three dimensional structures may lead to high computational costs, both in terms of memory requirements and CPU times required by the analysis.

In the implementation reported in this paper, only orthonormal Legendre polynomials are used to define the approximation bases. The parallel implementation of the hybrid-mixed stress model uses a message passing scheme based on the MPI standard [20]. The partitioning algorithm that was adopted consists in distributing in a weighted manner the number of elements per processor. According to the domain partition, each processor computes and stores locally, in a sparse format, the matrix entries associated to its sub-domain elements.

Concerning the solution scheme for the governing system in parallel, two distinct approaches were tested, namely the use of a direct method and a hybrid method. A third-party parallel direct solver code, named MUMPS [21,22], was used. This package is able to handle large sparse symmetric indefinite matrices. The hybrid solver is an in-house developed code highly inspired by the FETI method [23,24] and was developed to deal with the global governing system structure typical of the hybrid-mixed stress models.

The remainder of the paper is organized as follows. The fundamental equations are recalled first to establish the notation to be used. In Section 3 we shortly derive the hybrid-mixed stress finite element formulation and briefly describe some of the most distinctive features of the model. As a detailed description may be found in [2], only the essential aspects are presented here. Section 4 is devoted to the presentation of the relevant aspects concerning the parallel implementation of the numerical model. In Section 5 some numerical examples are shown in order to illustrate the effectiveness and performance of the parallel approach. Finally, conclusions are drawn in Section 6.

2. Fundamental relations

Let V denote the domain of the structure and let Γ_σ and Γ_u be the portions of the boundary Γ whereon the stresses, $\bar{\mathbf{t}}$, and the displacements, $\bar{\mathbf{u}}$ are prescribed, respectively.

In the equilibrium conditions,

$$\mathbf{D}\boldsymbol{\sigma} + \mathbf{b} = \mathbf{0} \quad \text{in } V \quad (1)$$

$$\mathbf{N}\boldsymbol{\sigma} = \bar{\mathbf{t}} \quad \text{on } \Gamma_\sigma \quad (2)$$

vectors $\boldsymbol{\sigma}$ and \mathbf{b} list the stress and body force components, and \mathbf{D} is the differential equilibrium operator. Eq. (2) expresses Cauchy's law in matrix form, where matrix \mathbf{N} collects the components of the unit outward normal to the boundary Γ_σ .

The compatibility conditions are written as follows,

$$\mathbf{e} = \mathbf{D}^* \mathbf{u} \quad \text{in } V \quad (3)$$

$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \Gamma_u \quad (4)$$

where vectors \mathbf{e} and \mathbf{u} collect the strain and the displacement components, respectively. When a geometrically linear behavior is assumed, the equilibrium and compatibility conditions, (1) and (3), represent conjugate transformations and \mathbf{D} and \mathbf{D}^* are adjoint operators.

The constitutive relations are written as follows, where \mathbf{f} is the symmetric non-singular matrix of elastic constants characterizing a linear reciprocal elastic law:

$$\mathbf{e} = \mathbf{f}\boldsymbol{\sigma} + \mathbf{e}_0 \quad (5)$$

The total strain is considered to be given by the sum of the elastic strain and the residual strain, \mathbf{e}_0 , the latter due to phenomena such as temperature changes.

The definition of the operators defined above for three dimensional elasticity problems may be found in references [16–18].

3. The three-field hybrid-mixed stress model (HM3)

The present model displays the distinguishing feature of using two independent approximations for the stress field, $\boldsymbol{\sigma}$, and the displacement field, \mathbf{u} , in the finite element domain, V , as follows:

$$\boldsymbol{\sigma} = \mathbf{S}_v \mathbf{X}_v \quad \text{in } V, \quad (6)$$

$$\mathbf{u}_v = \mathbf{U}_v \mathbf{q}_v \quad \text{in } V. \quad (7)$$

The displacements on the static boundary (which includes the boundaries shared by neighboring elements) are also independently approximated:

$$\mathbf{u}_\gamma = \mathbf{U}_\gamma \mathbf{q}_\gamma \quad \text{on } \Gamma_\sigma. \quad (8)$$

In this work, all field approximations are expressed in the form of a matrix–vector product. The matrices collect the adopted approximation functions, and the vectors their associated weights. These weights represent the generalized discrete form of the correspondent field. Of note is the fact that the approximation functions can be freely chosen as long as they are linearly independent and able to form a basis for the approximated field space. For this purpose, orthonormal Legendre polynomials are used [3,16,25], thus abandoning the more conventional nodal approximation approach. As a consequence, a direct physical interpretation of the problem variables is no longer possible, a minor drawback of the formulation.

Legendre polynomials can be generated recursively [26] as follows,

$$P_{n+1}(\xi) = 2\xi P_n(\xi) - P_{n-1}(\xi) - \frac{\xi P_n(\xi) - P_{n-1}(\xi)}{n+1} \quad (9)$$

with $P_0(\xi) = 1$ and $P_1(\xi) = \xi$, and where $P_n(\xi)$ represents the Legendre polynomial of degree n . This family of polynomials is orthogonal over the interval $[-1, 1]$,

$$\int_{-1}^1 \bar{P}_i(\xi) \bar{P}_j(\xi) d\xi = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases} \quad (10)$$

where

$$\bar{P}_n(\xi) = \sqrt{\frac{2n+1}{2}} P_n(\xi) \quad (11)$$

Fig. 1 depicts the first few members of the family of Legendre polynomials normalized according to (11).

The domain approximation functions are defined in the element local coordinate system (ξ, η, ζ) , in the form,

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