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The Variational Theory of Complex Rays applied to the shallow shell theory

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ABSTRACT

In the last few decades the interest of aerospace and automotive industries towards the study of the medium-frequency response of complex shell structure frames has grown. Recently some dedicated "wave" computational approaches have been developed. Among them, a Trefftz technique called Variational Theory of Complex Rays (VTCR) is catching on as an *ad hoc* method to deal with such vibration problems. This work presents the development of the VTCR in the shallow shell theory to increase its effectiveness and flexibility. First, general theory is given and two key properties of the solution demonstrated. After that, two numerical examples are deeply analyzed.

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1. Introduction

In recent years, the interest of aerospace and automotive industries has been focused on efficient virtual testing of the vibration response. Shallow shell structures are widely used in these industrial contests due to their high resistance and light weight. The equilibrium equations of shallow shells are quite complex and in almost every real case an analytic solution cannot be obtained. Thus, an effective method to predict vibrational behavior in shallow shell structures is needed. The Modal Overlap Factor [1] defines three zones: low, mid and high frequency range. The low-frequency range has been extensively studied by the Finite Element Method (FEM) [2] and the Boundary Element Method (BEM) [3]. On the other side, the high-frequency range can be addressed by the Statistical Energy Analysis (SEA) [1]. This technique neglects almost entirely spatial quantities to focus on global energy. This effective approach is based on some key assumptions assured in the high-frequency range. The medium-frequency range is still an open question. On one hand the FEM and BEM are not indicated in this frequency domain since the phenomena variation length is very small if compared to characteristic dimensions of the structure. For this reason the required number of Degrees of Freedom (DoFs) explodes [4]. On the other hand the SEA is not suggested because the key assumptions of the theory might be unsatisfied [5]. Notwithstanding a lot of work has been done to extend such theories to the medium-frequency range [6–8]. There are also methods developed specifically for the medium-frequency range such as the partition of unity method [9], the ultra-weak variational method [10], the asymptotic scaled modal analysis [11], the energy operator eigenmodes [12], Galerkin method [13], the wave boundary element method [14] or the wave-based method [15,16]. One of them is the Variational Theory of Complex Rays (VTCR). It approximates the vibrational problem solution with a sum of shape functions that identically satisfies the equilibrium equations while addressing the boundary conditions in weak form. This approach allows *a priori* independent approximations among subdomains. Thus, different (in number and type) shape functions can be chosen for each subdomain giving great flexibility to the method. It has already been applied to plate theory [17], to general shell theory [18], to transient dynamics [19], to 3D acoustic [20] and, on a wide frequency band [21,22]. Nevertheless the shell version of the VTCR can still be improved. Yet the in-plane inertia was not taken into account in previous works, the weak variational formulation must be customized for the specific geometry, and the VTCR formulation does not address the general case of a boundary (or a corner) shared by multiple subdomains. Such problems are analyzed and solved in this work; the in-plane inertia assumption is relaxed and two propagative waves that lead in-plane stresses and displacements are introduced. The customization phase of

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the weak variational formulation is avoided using the shallow shell approximation providing effectiveness and flexibility to the method. Since in this theory the surface geometry is projected to the underlying area, the tuning phase of the weak variational formulation is no more needed. Finally, a more general version of the VTCR is presented.

The present work is structured as follows: first, the general theory is proposed providing some useful properties. After, two numerical examples are presented. The first one is an academic case where the analytic solution is known. Convergence tests are performed and performances are compared with a FEM reference. The second one is a complex structure frame.

2. Theory

In this Section the equilibrium and the boundary equations are examined using the standard shallow shell approximations. The theory is akin to the one provided in [23,24]. After, some useful energy quantities are derived and the virtual work theorem is adapted for this specific case.

2.1. The equilibrium equations

The general reference example is presented in Fig. 1. The focus is on a generic subdomain Ω_i of the frame structure in Fig. 1. For the sake of clarity various boundary, corner, coupling, and surface conditions are split in Fig. 2. The $\partial_\Box\Omega_i$ symbol refers to a generic boundary of Ω_i where condition $\Box$ is applied. In the particular case of a boundary shared among subdomains, Γ is used instead. In the same way, for the conditions applied on corners, a symbol $\partial\partial_\Box\Omega_i$ is used. The generic corner shared among subdomains is indicated with $\mathcal{C}$. The over-line symbol $\overline{\Box}$ indicates that quantity $\Box$ is known (i.e. the value of the boundary constraint). The term $\hat{\mathbf{n}}_i$ is the normal unit vector of the boundary directed outward Ω_i. A subdomain is subject to loads, displacements constraints, and continuity conditions along the boundaries (Fig. 2a) and on the corners (Fig. 2b) as well as a distributed load per unit surface $\overline{g}_i$ (Fig. 2d). The displacement constraint $\overline{\mathbf{u}}_i = \left[\overline{\mathbf{v}}_i', \overline{w}_i\right]'$ along $\partial_{\overline{\mathbf{u}}_i}\Omega_i$ can be divided in in-plane $\overline{\mathbf{v}}_i$ and out-of-plane $\overline{w}_i$ components.[1] In the same way, the load per unit length $\overline{\mathbf{p}}_i = \left[\overline{\mathbf{b}}_i', \overline{q}_i\right]'$ along $\partial_{\overline{\mathbf{p}}_i}\Omega_i$ can be divided in in-plane $\overline{\mathbf{b}}_i$ and out-of-plane $\overline{q}_i$ components. The rotation condition $\overline{w}_{i,\hat{\mathbf{n}}_i}$ is imposed along $\partial_{\overline{w}_{i,\hat{\mathbf{n}}_i}}\Omega_i$ while a bending moment per unit length $\overline{m}_i$ is applied along $\partial_{\overline{m}_i}\Omega_i$. The corners of the subdomain are subject to out-of-plane displacements constraints $\overline{w}_{\mathcal{C}i}$ on $\partial\partial_{\overline{w}_{\mathcal{C}i}}\Omega_i$ and punctual forces $\overline{q}_{\mathcal{C}i}$ on $\partial\partial_{\overline{q}_{\mathcal{C}i}}\Omega_i$. Coupling conditions are applied on $\mathcal{C}$ and along Γ, in order to ensure continuity of stresses and displacements among subdomains (Fig. 2c).

All quantities of interest are defined in the complex domain. Each one is considered multiplied by $e^{j\omega t}$ where $j = \sqrt{-1}$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency and t is the time.

The geometry of the subdomain can be approximated by its projection on the local plane defined by the orthonormal basis $\{\hat{\mathbf{x}}_i, \hat{\mathbf{y}}_i, \hat{\mathbf{z}}_i\}$ and the displacement field can be restricted to (Kirchhoff's kinematics assumptions)

$$\mathbf{u}_i^z = \mathbf{u}_i - z_i\boldsymbol{\phi}_i$$

$$\boldsymbol{\phi}_i = \nabla w_i - \mathbf{R}_i \cdot \mathbf{v}_i$$

$$\mathbf{R}_i = \begin{bmatrix} \frac{1}{R_{xi}} & 0 \\ 0 & \frac{1}{R_{yi}} \end{bmatrix}$$

[1] $\Box'$ is the transpose operator.

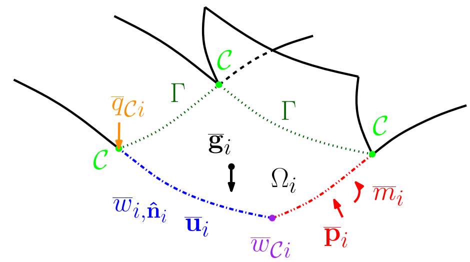

Fig. 1. Generic frame structure described in Section 2.1.

where $\mathbf{u}_i^z$ is the displacement thorough the thickness of the shell, $\mathbf{u}_i, \mathbf{v}_i$ and w_i are respectively the total, the in-plane and the out-of-plane displacements of the middle surface and $\mathbf{R}_i$ is the curvature matrix. $\mathcal{D}_i = \{\mathbf{u}_i, \mathbf{N}_i, \mathbf{M}_i\}$ is the set of fields that satisfies the equilibrium equations

$$\mathbf{u}_i^z \in \mathcal{U}_i^z \quad \text{finite energy displacement set,} \tag{1}$$

$$\{\mathbf{N}_i, \mathbf{M}_i\} \in \mathcal{S}_i \quad \text{finite energy generalized stress set,} \tag{2}$$

$$\nabla \cdot \mathbf{N}_i + \overline{\mathbf{g}}_{xyi} + \varrho_i h_i \omega^2 \mathbf{v}_i = 0 \quad \text{over} \quad \Omega_i, \tag{3}$$

$$\nabla \cdot (\nabla \cdot \mathbf{M}_i) + \mathbf{R}_i : \mathbf{N}_i + \overline{g}_{zi} + \varrho_i h_i \omega^2 w_i = 0 \quad \text{over} \quad \Omega_i, \tag{4}$$

$$\overline{\mathbf{g}}_i = \left[\overline{g}_{xi}, \overline{g}_{yi}, \overline{g}_{zi}\right]', \tag{5}$$

$$\overline{\mathbf{g}}_{xyi} = \left[\overline{g}_{xi}, \overline{g}_{yi}\right]', \tag{6}$$

$$\mathbf{M}_i = -D_i \mathbf{H}_i : \nabla\nabla w_i, \tag{7}$$

$$\mathbf{N}_i = \frac{12}{h_i^2} D_i \mathbf{H}_i : (\mathbf{E}_i - \mathbf{R}_i w_i), \tag{8}$$

$$\mathbf{E}_i = [\nabla \mathbf{v}_i]_{sym} = \frac{1}{2}(\nabla \mathbf{v}_i + \nabla \mathbf{v}_i'), \tag{9}$$

$$D_i = \frac{E_i h_i^3}{12(1 - \nu_i^2)}, \tag{10}$$

$$E_i = E_{0i}(1 + j\eta_i), \tag{11}$$

where $D_i\mathbf{H}_i$ is the Hooke's operator for plane stress, ϱ_i the density, η_i the damping coefficient, h_i the shell thickness, E_{0i} the Young modulus, ν_i the Poisson's ratio, $\Box : \Box$ the inner product matrix operator, and $\mathbf{N}_i$ and $\mathbf{M}_i$ are the stress and stress moment resultants tensors respectively. The sub-space of $\mathcal{D}_i$ associated with homogenized conditions ($\overline{\mathbf{g}}_i = 0$) is denoted as $\mathcal{D}_{0i} = \{\delta\mathbf{u}_i, \delta\mathbf{N}_i, \delta\mathbf{M}_i\}$. This definition will be useful in the next Sections.

2.2. The boundary conditions

In order to present a well-posed problem, three conditions imposed along each boundary and one on each corner are needed. The boundary and corner conditions presented in Fig. 1 can be classified in this way:

1. an in-plane condition, either a displacement constraint or a load per unit length ($\overline{\mathbf{v}}_i$ or $\overline{\mathbf{b}}_i$),
2. an out-of-plane condition, either a displacement constraint or a load per unit length ($\overline{w}_i$ or $\overline{q}_i$),
3. either a rotation or a bending moment per unit length ($\overline{w}_{i,\hat{\mathbf{n}}_i}$ or $\overline{m}_i$),
4. an out-of-plane condition, on corners either a displacement constraint or a punctual load ($\overline{w}_{\mathcal{C}i}$ or $\overline{q}_{\mathcal{C}i}$).

These boundary conditions can be imposed along edges and on corners shared among subdomains. On one hand, if the constraint is a displacement or a rotation and there is more than one subdomain involved there is no need of the relative coupling condition. In other words the subdomains can be considered decoupled for what concerns that particular constraint. On the other hand, if the condition is a load and there is more than one subdomain

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