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An exact block-based reanalysis method for local modifications

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ABSTRACT

This paper presents an alternative reanalysis algorithm based on the block matrix to address problems with local modifications. In this method, the modified stiffness can be classified as three parts: influenced region, stationary region and an interface region between them. The main computation cost concentrates on the influenced region by this specific blocked strategy. Compared with popular reanalysis methods, the proposed method can achieve an accurate response for large modification with lower computation cost. Several practical engineering problems are analyzed and the results are exactly as that performed by full analysis.

1. Introduction

Generally, although the modifications in each iterative design are local, the corresponding computational cost is expensive since the full analysis is commonly still repeatedly performed. Therefore, reanalysis techniques attempt to analyze structures efficiently and avoid full analysis after modifications. They have been successfully employed in various classes of structural reanalysis problems, e.g., such as vibration, nonlinear, dynamical. Considering the scale of changes, changes always can be classified as small, medium and large changes. Many suitable methods have been developed in recent decades.

For small changes, in the 1960s, Sherman and Morrison presented a direct method (DM), the method gives an exact solution where the change is only a given column or row of the original matrix [1].Woodbury proposed a Sherman–Morrison–Woodbury (SMW) formula, it is applicable to the situation where a relatively small proportion is changed, and the changes in the stiffness matrix could be represented by a small sub-matrix [2]. In 1988, the theorems of geometric variation were developed for the reanalysis of finite element structures when variations in the co-ordinates of the nodes of the elements are considered by Topping and Kassim [3]. Continuously, the theorems of geometric variation were applied into the nonlinear reanalysis problems [4]. In addition, the classical Taylor series and the binomial series expansion were utilized where the design variables have small changes. Akgün et al. proposed an approach that extended the low-cost method for low rank modifications to non-linear exact reanalysis along the spirit of the SMW formula [5]. Based on the sub-structuring technique, Kaveh and Fazli presented a simple and efficient graph method for the formation of cycle basis of the graph model, it is proposed for imposing the boundary conditions in the force method [6]. Pais et al. suggested an exact reanalysis algorithm based on an incremental Cholesky factorization, which could solve a linear system when a small portion of the coefficient matrix was modified [7].

For medium changes, in the early 2000s, a much smaller reduced system to approximate the response for a large system is suggested by Fox and Noor, respectively [8,9]. Fleury considered the second-order approximation and took the hybrid first–second order convex approximation strategies, the quality of results had a significant improvement [10,11]. Using polynomial fitting and response surface method (RSM), Haftka and Unal used simple functions to replace the response functions [12,13]. After the year 2000, based on the Neumann series expansion and epsilon-algorithm, Chen and Wu developed a new eigenvalue reanalysis method [14]. Xu et al. presented a method for the static reanalysis which uses super-linear convergence property of the Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton algorithm [15]. Chen et al. developed a new static displacement reanalysis method for structures by integrating perturbation and Padé approximation [16]. Yang et al. proposed an efficient multi-sample compression algorithm for elastoplastic nonlinear finite element method (FEM) [17]. Kaveh and Fazli used sub-structuring technique and modal approximations to reduce the size of the governing eigenproblem with slight perturbations of the regular structures [18]. Using the equilibrium equations and the singular value

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Nomenclature

S^0	influence domain
Ω	entire design space
Γ	interface boundary
S^m	modified influence domain
n_s	the given starting nodes in stationary region
n_i	the given starting nodes in influenced region
n_b	the given starting nodes in interface boundary
N_s,	the number of nodes in stationary region
N_i	the number of nodes in influenced region
N_b	the number of nodes in interface boundary

Matrices and vectors

$\mathbf{K}$	stiffness matrix
$\mathbf{R}$	load vector
$\mathbf{r}$	displacement vector
$\mathbf{L}$, $\mathbf{L}_s$, $\mathbf{L}_{c0}$, $\mathbf{L}_{cm}$	lower triangle matrices
$\Delta\mathbf{K}$	the changes of stiffness matrix
$\mathbf{W}$, $\mathbf{W}_1$	defined by Eq. (5)
$\mathbf{K}_d$	defined by Eq. (8)
$\mathbf{r}$i	basis vector
$\mathbf{r}_B$	the matrix of basis vectors
$\mathbf{B}$	defined by Eq. (14)
$\mathbf{K}_R$	reduced stiffness matrix
$\mathbf{R}_R$	reduced load vector
$\mathbf{y}$	coefficients vector
$\mathbf{A}$	a positive-definite square matrix
$\mathbf{S}$	stationary matrix
$\mathbf{U}$	upper-right of block-wise matrix
$\mathbf{V}$	lower-left of block-wise matrix
$\mathbf{D}$	lower-right of block-wise matrix
$\mathbf{K}_{c0}$	initial condensed matrix
$\mathbf{K}_{cm}$	modified condensed matrix
$\mathbf{R}_{c0}$	defined by Eq. (29)
$\mathbf{R}_{cm}$	defined by Eq. (40)

Subscripts and superscripts

0	subscript denotes initial structure
m	subscript denotes modified structure
$s(0)$	subscript denotes stationary region of initial structure
$s(m)$	subscript denotes stationary region of modified structure
$ii(0)$	subscript denotes influence region and interface boundary of initial structure
$ii(m)$	subscript denotes influence region and interface boundary of modified structure

decomposition (SVD), Kaveh et al. presented an efficient method for the analysis of those structures which can be formed by adding some members to the regular structures [19]. In addition, Rahami et al. gave an exact solution which is similar to sub-structuring method for solving irregular structures composed of regular and irregular parts [20]. Huang and Wang proposed a static reanalysis method without any auxiliary matrix operations called independent coefficient (IC) method for large-scale problems [21]. Based on the improved combined approximation (CA) method with shifting and matrix–matrix operations with Level-3 Basic Linear Algebra Subprograms (BLAS), Zheng et al. presented an efficient method for vibration reanalysis, which can calculate several eigenpairs of modified structure simultaneously [22]. Compared with these methods, the CA method might be the most popular one in recent years. The CA method was proposed by Kirsch in 1981 [23]. The advantage of the CA is that the efficiency of local approximations and the accuracy of global approximations are integrated. It has been proved to be feasible for static reanalysis, structural optimization, eigenvalue-problem, nonlinear analysis, dynamic reanalysis and sensitively analysis [24–28].

For large changes, in recent years, Chen presented an efficient method for determining the modified modal parameters in structural dynamic modifications [29]. Yang et al. suggested an adaptive static reanalysis method based on the Neumann series and epsilon algorithm when structural modifications are relatively large [30]. Zuo et al. proposed a new hybrid Fox and Kirsch's reduced basis method for structural static reanalysis [31]. Massa et al. focused on the modal reanalysis of structures subjected to multiple modifications of various origins, which could greatly affect the mode shapes [32]. He and Jiang suggested a new improved method for structural dynamic reanalysis with large changes in structural topology possessing added degrees of freedom (DOFs) in 2011 [33]. Song et al. proposed a novel direct reanalysis algorithm based on the binary tree characteristics to find updated triangular factorization for high-rank structural modifications [34]. Wang and Li, et al. proposed a parallel approximated inverse matrix based on symmetric successive over-relaxation (SSOR) and compressed sparse row (CSR) for real large scale problems [43].

To clarify the categories of reanalysis methods based on the scale of changes, different reanalysis methods and applications are shown in Table 1 Generally, most of reanalysis methods can achieve exact solutions for small changes, and predict approximations for medium-large modifications efficiently. For large changes, there is a considerable increase in computational effort when the higher order series are utilized. Furthermore, in order to guarantee the accuracy of approximations for repeated modifications, the adaptive strategy should be suggested to avoid accumulated errors.

The purpose of this study is to obtain accurate solution same as the solution obtained by FEM efficiently after modifications. The problem can be stated as shown in Fig. 1. Assuming the influence domain S^0 belongs to an entire design space Ω. Boundary Γ denotes the boundary between influence domain joined with the whole space Ω. After local modification, the corresponding influence domain S^m is changed, such as added hole, and the boundary Γ always keeps the same as the initial structure. In this study, we try to find an exact block-based (BB) method based on the block matrix inversion [35–39] and calculate the corresponding response after modifications efficiently and accurately. The method can be divided into offline and online stages. In the offline stage, the initial stiffness matrix can be partitioned into influenced region, stationary region and interface boundary between them. The block-wise matrix is calculated and influenced system is constructed which is unrelated to stationary region. In the online stage, modified stiffness matrix also can be classified into three corresponding regions. The calculations of block-wise matrix mainly concentrate on modified influenced region and boundary interface. Thus, the modified system can be considered as an updated influenced system. Furthermore, the BB method is still efficient for the local modifications when the percentage of influenced region is larger than 10%, and the accuracy keeps the same as the full analysis.

In the following sections, DM, CA and block matrix inversion are briefly introduced in Section 2. Basic theories and the details of the BB method are presented, and the corresponding performance validations are discussed in Section 3. A simple case and two practical examples are demonstrated in Section 4 and conclusions are given in Section 5.