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An extended finite element model for modelling localised fracture of reinforced concrete beams in fire

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ABSTRACT

A robust finite element procedure for modelling the localised fracture of reinforced concrete beams at elevated temperatures is developed. In this model a reinforced concrete beam is represented as an assembly of 4-node quadrilateral plain concrete, 3-node main reinforcing steel bar, and 2-node bond-link elements. The concrete element is subdivided into layers for considering the temperature distribution over the cross-section of a beam. An extended finite element method (XFEM) has been incorporated into the concrete elements in order to capture the localised cracks within the concrete. The model has been validated against previous fire test results on the concrete beams.

1. Introduction

Localised fracture of reinforced concrete members has recently been of interest to many researchers and engineers. Under fire conditions, reinforced concrete structural members (such as beams or slabs) are often forced into high deformation. This results in the formation of large individual cracks within the members, which has been observed in previous experimental tests [1–3]. These large individual cracks influence the exposure condition of the reinforcing steel bar to the fire. In some cases the steel reinforcements are directly exposed to fire, whereby significantly reducing the fire resistance of the structures. In some extreme cases, localised large cracks could even result in integrity failure of the structures [1–3]. A key factor in assessing the fire resistance of the structures is through predicting the localised fracture of their structural members. Recently, the performance-based approach has been used in the fire safety design of reinforced concrete structures, which requires the use of accurate numerical models for predicting the response of structural members in fire. In the past two decades, plenty of numerical simulations and analyses have been conducted for modelling concrete structures at elevated temperatures [4–14]. Those studies were all based on the continuum approach, in which smeared cracking was adopted to simulate the cracks within concrete members. Existing research indicates that models based on smeared cracking can predict global responses, such as deflection and structural stability, with reasonable accuracy. However, the smeared cracking model cannot capture the localised fracture within structural members, and quantitatively predict crack openings. As far as performance-based fire safety design is concerned, predicting the opening of individual cracks at critical sections of critical members can be a crucial issue when evaluating the reliability of structures under fire conditions. Little research has yet been done on modelling localised fractures for reinforced concrete structural members under fire conditions.

In the past, a discrete-cracking model has been used successfully for modelling the formation and propagation of cracks in structural members, when the crack path is known in advance. However, this approach has to limit the cracks to inter-element boundaries, which might cause mesh bias, or requires performing costly re-meshing during the analysis process. To model individual cracks more effectively, the extended finite element method (XFEM) was introduced [15,16], based on the partition of unity theory [17]. The XFEM approaches in conjunction with cohesive-zone models [18–21] allow displacement jumps within conventional finite elements to analyse localisation and fracture in engineering materials. In the last decade, the XFEM has been successfully extended to many applications, such as multiple cracks in brittle materials, intersecting cracks and dynamic crack growth [22–26]. In terms of computer implementation of enriched finite element methods, a general structure for an object-oriented enriched finite element code (the XFEM library) was presented by [27], which had been designed to meet all natural requirements for modularity, extensibility, and robustness. Another open-source software framework called PERMIX for multiscale modelling of material

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NOTATION

$\mathbf{B}^{u}_{sta}$	regular strain–displacement transformation matrix
$\mathbf{B}^{a}_{enr}$	enhanced strain–displacement transformation matrix
$\mathbf{D}$	material constitutive matrix of plain concrete
$\mathbf{f}^{int}$	element internal force vector
$\mathbf{f}^{int}_{a}$	enhanced element internal force vector
$\mathbf{f}^{int}_{u}$	regular element internal force vector
$\mathbf{f}^{int}_{\Gamma}$	element internal force vector corresponding to traction
G_f	fracture energy of concrete
$\mathbf{K}_{aa}$	enhanced element stiffness matrix
$\mathbf{K}_{uu}$	regular element stiffness matrix
$\mathbf{K}_{\Gamma}$	element stiffness matrix corresponding to traction
$\mathbf{T}_{a}$	tangent stiffness of traction–separation relation
$\mathbf{t}_{a}$	traction within the cracks
$sign(x)$	sign function
$\mathbf{u}_{\mathrm{cont}}$	vector of continuous displacement field
$\mathbf{u}_{\mathrm{dis}}$	vector of discontinuous displacement field
$\Psi_i(\mathbf{x})$	enhancement function

failure was presented by [28]. The integration method for the XFEM based on Schwarz–Christoffel mappings was proposed by [29] to simplify the numerical integration on arbitrary polygonal domains. The application of strain smoothing in finite elements was extended to the extended finite element method to form the smoothed extended finite element method (cell-based smoothed XFEM, edge-based smoothed XFEM, and node-based smoothed XFEM) [30–32]. By transforming interior integration into boundary integration, strain smoothing simplifies the integration of discontinuous approximations of the XFEM and suppresses the need to integrate singular functions numerically. The smoothed XFEM is insensitive to mesh distortion and locking and could be a competitive alternative to solve complex 3D problems. The strain smoothing method was extended to higher-order elements by [33], and it also concluded that the method is only beneficial when the enrichment functions are polynomial. Besides the XFEM, there are also some alternate approaches for modelling the strong discontinuity. The numerical results of the embedded finite element method (EFEM) and XFEM were compared in [34], and various methods for numerical modelling of multifield fracturing, such as interface and embedded discontinuity elements, XFEM, thick level set and phase field models, and a discrete crack approach with adaptive remeshing, were discussed in [35]. Recently, the meshfree method based on a partition of unity concept was also developed to model concrete and more general non-linear materials [36–38]. This method has been used to successfully model the reinforced concrete structural members at the ambient temperature [39,40], where a coupled particle-finite element approach was adopted and the reinforcement was coupled with the concrete via a ‘barscale’ bond model for modelling the pullout and splitting failure. However, so far, limited efforts have been made to use the XFEM in modelling reinforced concrete structural members in fire.

The main objective of this paper is to develop a robust finite element procedure for modelling the localised fracture of reinforced concrete members in fire conditions. The model developed can be used for structural fire engineering design of reinforced concrete beams and enable engineers to assess both the structural stability (global response) and the integrity (localised fracture) of the beams. In the past, the majority of reinforced concrete beams at elevated temperatures have been simulated by the conventional finite element method, in which the generalised isoparametric elements have usually been used. In the procedure proposed in this paper the isoparametric elements are still employed so that relatively small modifications of the available finite element model are required. The new procedure could be easily applied to the fire design for practical building structures. In this paper a 2D model is used to model reinforced concrete beams. Since mesh distortion and locking are not the main concerns within the scope of this paper, only the straight crack is considered and the standard XFEM formulations and numerical integration procedure are employed. In this new model a reinforced concrete beam is represented as an assembly of 4-node quadrilateral plain concrete, 3-node main reinforcing steel bar, and 2-node bond-link elements. The extended finite element method (XFEM) is incorporated into plain concrete elements in order to capture the localised cracks of concrete within the member. The original contributions of the model presented in this paper are:

- Combine the XFEM plain concrete element with the reinforcing bar element and bond-link element successfully. Due to the bond-link element and plain concrete element sharing the same node, one important issue with which the model should deal is the compatibility of nodal displacements referenced to the plain concrete element and bond-link element. This is due to the nodal displacement of the cracked XFEM plain concrete element being divided into two parts: continuous part and discontinuous part. These displacements are not compatible with the nodal displacement of the bond-link element. Therefore, a special shifted enhancement function is used in order to obtain the total nodal displacement (including both continuous and discontinuous parts) of the cracked XFEM plain concrete elements. This satisfies the compatibility of the nodal displacements of both the XFEM plain concrete element and bond-link element.
- With the help of the bond-link element and steel bar element, the developed model has the capability to consider the influence of the bond characteristic between the concrete and reinforcing steel bar on the initiation and propagation of each individual crack within the reinforced concrete beam. Due to the influence of the reinforcing steel bar, the Newton–Raphson iteration procedure can be employed to solve this very nonlinear problem up to the failure of the whole beam. This is significantly different with conventional XFEM models, in which a complex solution procedure needs to be developed.
- Even for the adoption of a 2D model for modelling plain concrete, the model developed in this paper is still complex, because the effects of temperatures induced by fire need to be taken into account. The XFEM plain concrete elements are subdivided into layers for considering the temperature distribution over the cross-section of a beam. Since the temperature varies across different layers, a robust criterion has been developed to determine the initiation of individual cracks within the XFEM plain concrete elements. Moreover, the complications of structural behaviour in fire, such as thermal expansion, degradation of bond characteristics between a reinforcing steel bar and concrete, and the change of material properties with temperature, are modelled.

The new model has been validated against some previous fire tests of reinforced concrete beams. It is clear that the developed nonlinear procedure proposed in this paper can predict cracking patterns (flexural cracks and shear cracks) of the reinforced concrete beams properly. The model is capable of predicting the global response of reinforced concrete beams in fire with good