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A guided Bayesian inference approach for detection of multiple flaws in structures using the extended finite element method

Gang Yan [a], Hao Sun [b], Haim Waisman [b,*]

[a] State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
[b] Department of Civil Engineering and Engineering Mechanics, Columbia University, New York, NY 10027, USA

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ABSTRACT

We propose a guided Bayesian inference approach for detection and quantification of multiple flaws in structures without *a priori* knowledge on the number of flaws. Uncertainties due to modeling errors and measurement noise are explicitly considered in the Bayesian framework. Flaws are approximated by circular-shaped voids that can be easily represented by a set of parameters including the center coordinates and the radii. The extended finite element method (XFEM) is employed as the forward solver in the inverse detection framework, where re-meshing requirements in the vicinity of the flaws are completely alleviated. By comparing the measurement data and the output of the XFEM forward model, Bayes' theorem is used to update the probability distributions of the flaw parameters, leading to a full statistical quantification of flaws. Since the number of flaws is unknown beforehand, a trans-dimensional reversible jump Markov chain Monte Carlo (RJMCMC) algorithm is employed for sampling the posterior distributions of flaw parameters within the varying parameter spaces. The RJMCMC algorithm is guided by predefined prior information which is based on damage indices defined at each sensor location. These indices are obtained by comparing the undamaged and damaged measurement states. Numerical studies are carried out to demonstrate the effectiveness of the proposed statistical multiple-flaw quantification method.

1. Introduction

The increasing emphasis on integrity of critical structures such as aircrafts, civil infrastructures, and pressure vessels, urges the needs to monitor structures so as to detect flaws at an early stage to prevent catastrophic failure. During the past two decades, with the advances in the area of structural health monitoring (SHM), a wide range of techniques for flaw detection and quantification have been proposed and validated numerically and/or experimentally [1–3]. Among these developments are model-based methods, which can quantitatively identify the location and size (extent) of flaws and have drawn significant attention [4–15].

Mathematically speaking, model-based methods can be viewed as a nonlinear inverse process, in which flaw parameters of the model are iteratively updated to match the structural response. The response of the structure is usually captured by a set of sensors. Hence, the problem is often translated into an optimization problem with the objective to minimize the discrepancies between the forward model output and the sensor measurement.

When modeling the forward problem, the traditional finite element method (FEM) is a straightforward tool [4–7]. It has been widely used in model-based damage detection for discrete truss and frame structures using either static or dynamic responses. However, for continuum structures like plates, FEM involves the issue of remeshing the computational domain to conform the boundaries of flaws at each updating step, which is a non-trivial and time-consuming task, especially for complex flaws and their host media. One alternative to overcome the costly remeshing tasks is to employ the boundary element method (BEM), which has also been employed for the forward analysis in solving the inverse flaw quantification problems [8–11]. BEM transforms the governing equations of elasticity into a boundary integral equation and solves the problem using Green's function. Nonetheless, if the flaw boundaries are arbitrarily shaped, calculation of the Green's function within a heterogeneous solid becomes quite difficult, limiting the use of BEM. Other modeling techniques are the spectral element method [12–14] and the strip element method [15], which have been occasionally used, but they are only confined to simple structures and flaw configurations. Only recently, a new numerical approach called the extended finite element method (XFEM) was proposed to model problems with strong/weak discontinuities

* Corresponding author.
E-mail address: waisman@civil.columbia.edu (H. Waisman).

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[16,17]. XFEM provides an attractive alternative to standard FEM in that it does not require fine spatial resolution in the vicinity of discontinuities nor does it require repeated re-meshing to properly address propagation of cracks or detection of flaws in such inverse problems.

From an inverse optimization aspect, one can usually consider two main categories of methods: gradient based algorithms and heuristic algorithms which do not require gradients to evolve the parameters. Gradient methods, such as Newton-type methods, have the advantage of fast convergence to a solution provided that the initial guess is close enough to the solution. Nevertheless, in such inverse problems it is hard to determine a suitable initial guess and these methods may converge to a local minimum which is not the optimal solution. Another drawback of gradient methods is that one needs to know the number of optimization variables (flaw parameters in our case) beforehand. However, since the number of flaws is unknown to begin with, gradients cannot easily be defined and some adaptive procedure is required which makes the application of such methods difficult [4]. Heuristic algorithms such as genetic algorithms (GA) and other evolutionary methods may alleviate the limitations of gradient based methods and have widely been adopted for flaw detection problems [11]. These methods can easily be integrated into existing codes and offer powerful global search capabilities [6,8,9,13]. Nonetheless, these methods are also limited in that they converge much slower than gradient based methods and if the number of optimization variables is large, significant computational power is required to evolve these methods in a reasonable time. To this end, several researchers have proposed to combine heuristic algorithms with gradient based methods to obtain fast convergence to a global minimum [15].

Recently, Rabinovich et al. [18,19] proposed a new numerical technique based on XFEM and GA for crack identification in a flat linear membrane using time-harmonic responses. This scheme takes advantage of XFEM that alleviates the remeshing burden in each forward analysis of the optimization process. The later work by Waisman et al. [20] extended this XFEM-GA approach to quantify various types of flaws such as straight cracks, circular holes, and irregular-shape holes. This technique is further improved by Chatzi et al. [21] by proposing a generic parametric enrichment function (i.e., using ellipses) which is utilized to detect any type of flaws (cracks or holes) of any shape. Jung et al. [22,23] performed a study on the identification of a scatterer embedded in an elastic heterogeneous media using dynamic XFEM, where the inverse problem is casted as a minimization problem whereby the unknown shape parameters of the scatterer are updated by a gradient-based search algorithm. Nanthakumar et al. [24] extended the XFEM-based flaw detection scheme to piezoelectric structures using an "intermediate" optimization algorithm called the multilevel coordinate search (MSC) method. While these XFEM-based approaches only consider single flaw detection problems, Sun et al. [25] proposed a topological variable-based enhanced artificial bee colony (EABC) algorithm to quantify multiple flaws by using XFEM as forward solver. This algorithm introduces topological variables into the search space as a part of optimization, which is used to adaptively activate/deactivate flaws during the run time until convergence is reached. Later on, Sun et al. [26] proposed a novel multiscale algorithm based on XFEM for nondestructive detection of multiple flaws. The key idea of this approach is to apply a two-step optimization scheme, where first rough flaw locations are quickly determined and then fine tuning is applied in these localized subdomains to obtain global convergence to the true flaws. Nanthakumar et al. [27] proposed an innovative methodology to detect multiple flaws in piezoelectric structures using XFEM and topology optimization. This method used a combination of classical shape derivative and the level-set method to minimize the cost function. The void configuration does not require external parameterization as it is implicitly represented by level sets. It is effectively able to determine the number of voids and its corresponding locations and shapes.

Up to date, most of the inverse problems formulated for flaw quantification in literature are solved deterministically (even though the search mechanisms of the heuristic algorithms, such as GA, are stochastic, most of their results are reported deterministically), in which uncertainties from modeling errors, measurement noise, and other influencing sources are not explicitly considered. However, these uncertainties are unavoidable in practical flaw detection. Hence, probabilistic and statistical approaches are sometimes more appropriate than deterministic approaches since probability distributions can be used to quantify the various uncertainties in this process. In particular, the Bayesian statistical framework has been established and applied to structural system identification by Beck and his colleagues [28–31], and then extended to various structural damage identification scenarios [7,14,32–34]. One outstanding advantage of the Bayesian approach is that engineering judgements or expert knowledge can be easily incorporated into the process as the prior information to reduce possible uncertainties. In addition, rather than pinpointing a single solution by deterministic approaches, the Bayesian approach can provide the probability distribution of the unknown parameters, giving both point and interval estimates. This is very important and useful as pointed by Beck [35] "there are really no true values of the parameters to estimate because any chosen model gives only an approximation of the real system behavior".

The aim of this work is to combine the Bayesian framework with XFEM to provide a statistical approach for nondestructive multi-flaw identification considering uncertainties from modeling errors and measurement noise. Specially, a trans-dimensional reversible jump Markov chain Monte Carlo (RJMCMC) method [36] is employed to draw the posterior distributions of the flaw parameters due to the missing knowledge of the number of flaws. In addition, a pre-analysis procedure based on damage indices defined at each sensor location is introduced to provide prior information to guide the RJMCMC algorithm for better convergence. The paper is structured as follows. Section 2 describes the main idea of the trans-dimensional Bayesian approach for flaw detection and quantification. In Section 3, a brief introduction of RJMCMC and its application for quantification of the uncertain flaw parameters are presented. The XFEM for forward analyses is described in Section 4. To verify the proposed method, numerical examples with increasing level of difficulty are illustrated in Section 5. Finally, concluding remarks are given in Section 6.

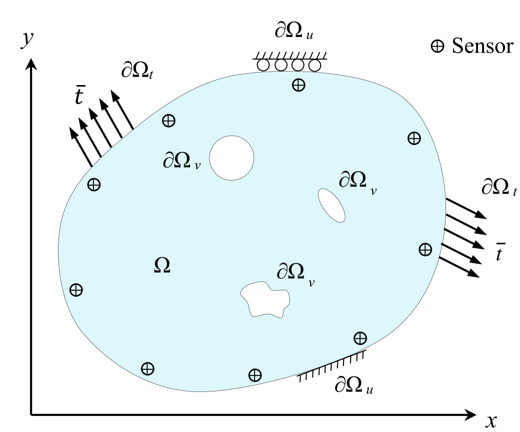

Fig. 1. A generic solid structure with traction-free void flaws.