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In-plane free vibration and response analysis of isotropic rectangular plates using the dynamic stiffness method

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ABSTRACT

The development of the dynamic stiffness matrix for isotropic rectangular plate with arbitrary boundary conditions undergoing in-plane free vibration is presented in this paper. Gorman's superposition method is exploited to obtain the solution of the governing equations of motion. The dynamic stiffness matrix is derived using the Projection method. The obtained results are in good agreement with the results obtained using exact solutions for some special cases and also with finite element solutions. Modeling of plates undergoing in-plane vibration using dynamic stiffness method demonstrate high precision, accuracy and low memory requirement in comparison with the finite element method.

1. Introduction

Plates are widely used in engineering practice as components of different structures such as airplane wings, building walls, and floor slabs. These structures are sometimes designed to experience low frequency excitations such as wind and earthquake. However, there is a growing need for reliable prediction of the dynamic response of structures due to high frequency excitation, such as acoustics, blasts and impacts. Therefore, it is important to apply an accurate and efficient method for dynamic response analysis of such structures.

For structural modeling, the conventional finite element method (FEM) [1] is most frequently used. The mesh size of finite elements depends on the highest frequency in the analysis. Increasing the number of finite elements takes greater time and effort to solve the problem. In addition, the results become unreliable especially for high frequencies. As an alternative to the FEM, the Dynamic Stiffness Method – DSM (or the Spectral Element Method – SEM) has been used in the literature to analyze a wide range of dynamic problems [2–5]. The DSM is based on harmonic representation of the displacement field and on the exact solution of the governing differential equations of motion defined in the frequency domain [2]. Consequently, the dynamic stiffness matrix is frequency dependent. The combination of uniform mass distribution, minimal discretization of the domain, simple assembly procedure (as in the FEM) and efficient Fast Fourier Transform (FFT) algorithms makes the DSM a powerful tool for solving various dynamic problems.

The DSM is well developed for structures consisting of one-dimensional elements (beams and bars) and Levy-type plates for which the exact solutions of the governing differential equations of motion exist. The corresponding dynamic stiffness matrices have been derived in the explicit form. Therefore, only one element can exactly represent the dynamic behavior at any frequency. For two-dimensional elements such as plates it is difficult to obtain exact solutions of the governing equations of motion that satisfy all boundary conditions. To find a solution of the problem, plate displacements are generally presented in infinite series form. For practical purposes, the series is truncated, which introduces some error. Consequently, the solutions are somewhat approximate, but may satisfy the prescribed degree of accuracy. Additionally, plate displacements along the boundary are continuous functions of spatial variables. Therefore, it is not possible to define the relation between the displacement and force vectors on the free plate boundary, unlike the one-dimensional elements and Levy-type plates. The spatial dependence can be avoided by using the Projection method [6,7], which is based on the projection of the displacements and forces on the boundary onto a set of trigonometric functions.

The research in the field of two-dimensional dynamic stiffness elements has mainly been directed toward the free transverse vibration analysis of isotropic plates with two opposite edges simply supported, [8,9]. Boscolo and Banrjee [10,11] made a step forward and explicitly developed the dynamic stiffness matrix of composite laminate Levy-type plate using classical and first order shear deformation theory. Kulla [12] was the first who developed

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a high precision continuous element for transverse vibration of plates with arbitrary boundary conditions and applied it in the dynamic response analysis. Casimir et al. [7] built the dynamic stiffness matrix of two-dimensional Kirchhoff's plate element with free edges using Gorman's superposition method [13] and the Projection method and computed the dynamic response of completely free plate subjected to point harmonic load.

Unlike the free transverse vibration, only a few studies have been dealt with the in-plane free vibration of rectangular plates. Exact solutions of free in-plane vibration and the corresponding dynamic stiffness matrices can be found in the literature only for Levy-type and two-edge infinite plates. Anderson et al. [14] developed the dynamic stiffness matrix of a two-edge plate element, and incorporated it in the computer program VICON for exact buckling and free vibration analysis of plate assemblies such as stiffened panels, open and box section members. Danial et al. [15] derived the dynamic stiffness matrix for two-edge infinite plates for both transverse and in-plane vibrations and applied it in the dynamic response analysis of multiple connected plates oriented at arbitrary angles. Bercin [16] exploited the dynamic stiffness method to calculate free vibration characteristics of various simply supported, stiffened and directly orthogonally coupled rectangular plate assemblies. Boscolo and Banerjee [17] investigated the in-plane free vibration behavior of plates using the DSM. They explicitly developed the dynamic stiffness matrix of a plate element with two opposite edges simply supported while arbitrary boundary conditions have been assigned to other edges. They also detected some natural frequencies that had been missed in the previous works [14]. Gorman [18] developed an analytical-type solution using the superposition method to obtain free in-plane vibration characteristics of completely free rectangular plate. Gorman's analytical method is applicable for free in-plane vibration analysis of an individual plate with arbitrary boundary conditions and cannot be easily extended to analyze complex structures like plate assemblies consisting of plates of variable geometrical and material properties.

In the present research a general method for free in-plane vibration and dynamic response analyses of rectangular plates having arbitrary boundary conditions, geometrical properties and non-uniform distribution of structural materials has been presented. Gorman's superposition method [18] has been exploited here in order to obtain the solution of the governing equations of motion. Frequency dependent matrices **D** and **F** which relate the displacement and force vectors through the constant vector **C**, have been derived explicitly for the first time in Section 2.2 using the Projection method [6]. The dynamic stiffness matrix of a plate element cannot be developed in the explicit form, since the size of the dynamic stiffness matrix is directly influenced by the number of terms used in the general solution. Therefore, the elements of the dynamic stiffness matrix have been evaluated numerically. The global dynamic stiffness matrix of plate assemblies with arbitrary boundary conditions has been constructed using the same assemblage procedure as in the FEM.

Finally, in Section 3 results of several numerical simulations using the proposed method are validated against the exact solutions available in the literature [18,19], the results based on Raylegh–Ritz method [20] and Finite Element Software SAP2000 [21].

2. Dynamic stiffness elements for in-plane vibration analysis

2.1. General solution

A rectangular plate element with coordinate system and in-plane displacements $u(x,y,t)$ and $v(x,y,t)$, and forces $N_x(x,y,t), N_y(x,y,t)$ and $N_{xy}(x,y,t)$, is shown in Fig. 1a. The equations of motion obtained from the equilibrium of forces, can be written as [17]:

$$\begin{aligned} &\frac{\partial^2 u}{\partial x^2} + a_1\frac{\partial^2 u}{\partial y^2} + a_2\frac{\partial^2 v}{\partial x \partial y} - \frac{\rho h}{D}\frac{\partial^2 u}{\partial t^2} = 0, \\ &\frac{\partial^2 v}{\partial y^2} + a_1\frac{\partial^2 v}{\partial x^2} + a_2\frac{\partial^2 u}{\partial x \partial y} - \frac{\rho h}{D}\frac{\partial^2 v}{\partial t^2} = 0, \end{aligned} \tag{1}$$

where

$$a_1 = \frac{1-\nu}{2}, \quad a_2 = \frac{1+\nu}{2}, \quad D = \frac{Eh}{1-\nu^2}, \tag{2}$$

h is plate thickness, ρ is mass density, E is the Young's modulus, ν is the Poisson's ratio of the plate material. In-plane forces in terms of plate displacements are given as:

$$\begin{aligned} &N_x(x,y) = D\left(\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y}\right), \\ &N_y(x,y) = D\left(\nu\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right), \\ &N_{xy}(x,y) = Da_1\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \end{aligned} \tag{3}$$

Introducing harmonic representation of the in-plane displacements as:

$$\begin{aligned} &u(x,y,t) = \hat{u}(x,y,\omega)e^{i\omega t}, \\ &v(x,y,t) = \hat{v}(x,y,\omega)e^{i\omega t}, \end{aligned} \tag{4}$$

where ω is angular frequency, $\hat{u}(x,y,\omega)$ and $\hat{v}(x,y,\omega)$ are the amplitudes of displacements in the frequency domain, the Fourier transform of Eq. (1) can be expressed as:

$$\begin{aligned} &\frac{\partial^2 \hat{u}}{\partial x^2} + a_1\frac{\partial^2 \hat{u}}{\partial y^2} + a_2\frac{\partial^2 \hat{v}}{\partial x \partial y} + \frac{\omega^2}{c_p^2}\hat{u} = 0, \\ &\frac{\partial^2 \hat{v}}{\partial y^2} + a_1\frac{\partial^2 \hat{v}}{\partial x^2} + a_2\frac{\partial^2 \hat{u}}{\partial x \partial y} + \frac{\omega^2}{c_p^2}\hat{v} = 0, \end{aligned} \tag{5}$$

where

$$c_p = \sqrt{\frac{E}{\rho(1-\nu^2)}} \tag{6}$$

represents the longitudinal wave speed.

According to Gorman's superposition method [18], the in-plane displacements of a rectangular plate can be split into four contributions: symmetric–symmetric (SS), anti-symmetric–anti-symmetric (AA), symmetric–anti-symmetric (SA) and anti-symmetric–symmetric (AS):

$$\begin{aligned} &\hat{u}(x,y) = \hat{u}_{SS}(x,y) + \hat{u}_{AA}(x,y) + \hat{u}_{SA}(x,y) + \hat{u}_{AS}(x,y), \\ &\hat{v}(x,y) = \hat{v}_{SS}(x,y) + \hat{v}_{AA}(x,y) + \hat{v}_{SA}(x,y) + \hat{v}_{AS}(x,y). \end{aligned} \tag{7}$$

The free vibration mode is symmetric about an axis if displacement normal to this axis has a symmetric distribution about it. For each contribution defined in Eq. (7) the solution of the system of Eq. (5) is obtained superimposing the solutions that correspond to the Levy-type solutions, [18].

2.1.1. Symmetric–symmetric contribution (SS)

In this case the displacement $\hat{u}_{SS}(x,y)$ will have symmetric distribution about the y axis and displacement $\hat{v}_{SS}(x,y)$ will have symmetric distribution about the x axis. In order to satisfy the defined double symmetry condition, as well as the system of Eq. (5), the in-plane displacements are represented as:

$$\begin{aligned} &\hat{u}_{SS}(x,y) = {}^1\hat{u}_{SS}(x,y) + {}^2\hat{u}_{SS}(x,y), \\ &\hat{v}_{SS}(x,y) = {}^1\hat{v}_{SS}(x,y) + {}^2\hat{v}_{SS}(x,y), \end{aligned} \tag{8}$$