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Temporal stabilization nodal integration method for static and dynamic analyses of Reissner–Mindlin plates

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ABSTRACT

In this paper, a temporal stabilized nodal integration method (sNIM) using 3-node triangular elements is formulated for elastic-static, free vibration and buckling analyses of Reissner–Mindlin plates. Two stabilization terms are added into the smoothed potential energy functional of the original nodal integration, consisting of squared-residual of equilibrium equations. A gradient smoothing technique (GST) is used to relax the continuity requirement of shape function. The smoothed Galerkin weak form is employed to create discretized system equations, and the node-based smoothing domains are formed to perform the smoothing operation and the numerical integration. A stabilization parameter is finally introduced to the modified system for the sake of curing temporal instability. Numerical tests provide an empirical value range of stabilization parameter, within which very accurate and stable results can be obtained for both static and eigenvalue problems.

1. Introduction

Plates are the most widely used structural components in civil, mechanical and aerospace engineering. Static, free vibration and buckling analyses of plate structures play an increasing important role in their engineering applications. Due to the limitations of analytical methods, the finite element method (FEM) is still the most powerful numerical tool to simulate behaviors of plates so far.

In the practical applications, lower order Reissner–Mindlin plate elements are preferred due to their simplicity and efficiency. They require only C^0-continuity requirement for both translational and rotational displacement fields. However, the development of lower-order plate elements in the thin limit often suffer from the shear-locking phenomenon due to incorrect transverse forces under bending. To overcome this difficulty, a large amount of efficient work have been done by researchers and various elements have been proposed such as reducing integration [1] or selective integration [2], discrete Kirchhoff triangular (DKT) elements [3–5], Enhanced Assumed Strain (EAS) methods [6–8], Assumed Natural Strain (ANS) methods [9–11] and discrete shear gap (DSG) method [12]. All these methods are of great useful in reducing the shear-locking deficiency and increase the solution accuracy to some extend.

On an other front of development of numerical methods, meshfree or meshless method has attract more and more scholars' attention. Belytschko et al. [13] first proposed a so-called Element Free Galerkin (EFG) method based on the moving least-square (MLS) approximation, which laid foundations for the subsequent studies. Later, researchers extended the EFG for plate and shell analyses [14,15] and very reasonable results were obtained. Meshless local Petrov–Galerkin (MLPG) method [16–18] is another attractive meshless method used in engineering analyses. Based on MLPG, a large amount of works have been done by scholars [19–21]. Unfortunately, the MLS shape function does not possess the Kronecker delta function properties, thus various improvements have been developed to overcome this deficiency. One of the remarkable achievements is the point interpolation method (PIM) [22] or radial point interpolation method (RPIM) [23,24], in which the shape functions possess the Kronecker delta function properties and the essential boundary conditions can be easily imposed. Later, researchers successfully applied this method to the static and dynamic [25–27], as well as the buckling analyses of plates [28,29]. As one of the most influential meshfree methods, Liu et al. [30–32] proposed a so-called reproducing kernel particle method (RKPM) in 1995, which can be seen as a good reference of our research in this work. The following extensive and in-depth research based on RKPM can be found in JS Chen's group [33–35]. In their work, they put forward a stabilized conforming nodal integration method, which lead the development of nodal integration on meshless methods in the following years.

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In the effort to further advance finite element method (FEM), Liu et al. have extended the concept of smoothing domains to formulate a family of smoothed finite element method (SFEM) [36,37]. Their work on SFEM combines the advantages of both FEM and meshfree method, and is considered a worth investigating for further study. Based on the concept of SFEM, researchers developed the so-called edge-based smoothed finite element method (ES-FEM) [38–40] and node-based smoothed finite element method (NS-FEM) [41,42]. The other works of plates and shell analyses using SFEM include those given in [43,44].

In the development of nodal integration, two types of instability, spatial instability and temporal instability, have been found. Through the smoothing operation, the spatial instability can be successfully eliminated, while the temporal instability is still an open issue so far. A spatially stable model always produces a unique and convergent solution for static problems when functions are bounded. However, this does not guarantee a stable solution for dynamic problems. In [45], Beissel and Belytschko pointed out that the nodal integration suffers from spurious single modes due to underintegration of the weak form, and treats it by the addition to the potential energy functional of a stabilization term which contains the square of the residual of the equilibrium equation. Bonet and Kulasegaram [46] presented a least-squares stabilization procedure to cure temporal instability of nodal integration in metal forming simulations. Based on these previous works, Zhang and Liu [47] further developed a stabilization procedure for NS-FEM, and then provided a recommended range for the stabilization parameter.

In this work, the temporal stabilization nodal integration is further extended to the analysis of Reissner–Mindlin plates. Three-node triangle, which can be generated automatically for complicated geometries, is chosen as the background cells. In order to apply the squared-residual stabilization technique to the original nodal integration using linear triangular elements, the gradient smoothing technique (GST) is extended to the second order derivatives, so that only the first order derivatives of the shape function are needed in our formulation. The discretized system equations are derived according to the smoothed Galerkin weakform. A stabilization parameter is finally introduced to the further formed stiffness matrix. Numerical tests provide an empirical value range of stabilization parameter, within which very reasonable results can be obtained for both elastic-static and eigenvalue problems.

2. Theoretical formulations

2.1. Basic equations for Reissner–Mindlin plate

In this section, the basic equations of Reissner–Mindlin plate are briefed. Let us consider a plate under bending deformation.

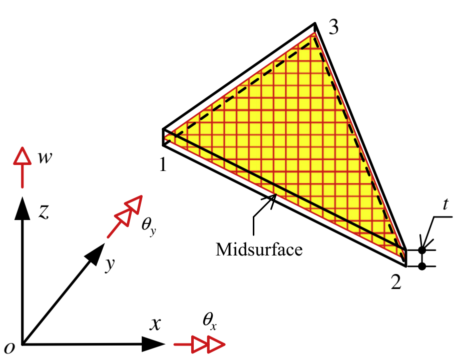

Fig. 1. 3-node triangular element and positive directions of the deflection and two rotations.

The middle surface of the plate is chosen as the reference plane that occupies a domain $\Omega \subset R^2$ as shown in Fig. 1. Let w and $\boldsymbol{\theta}^T = (\theta_x, \theta_y)$ be the transverse displacement and the rotations about the x and y axes, respectively. Then the unknown vector of three independent field variables at any point in the problem domain can be given as

$$\mathbf{u}^T = \{ w \quad \theta_x \quad \theta_y \} \tag{1}$$

Let us assume that the material is homogeneous and isotropic with Young's modulus E and Poisson's ratio ν. The governing differential equations of the static Reissner–Mindlin plate are

$$\nabla \cdot \mathbf{D}_b \boldsymbol{\kappa}(\boldsymbol{\theta}) + \chi t G \boldsymbol{\gamma} = 0 \text{ in } \Omega \tag{2a}$$

$$\chi t G \nabla \cdot \boldsymbol{\gamma} + p = 0 \text{ in } \Omega \tag{2b}$$

$$w = \bar{w}, \quad \boldsymbol{\theta} = \bar{\boldsymbol{\theta}} \text{ on } \Gamma = \Gamma_t \tag{2c}$$

in which $\mathbf{D}_b$ is the bending stiffness constitutive matrix, G is the shear modulus, $\chi = 5/6$ is the shear correction factor, t is the plate thickness, $p = p(x,y)$ is a distributed load per unit area, $\boldsymbol{\kappa}$ and $\boldsymbol{\gamma}$ are the bending and shear strains, respectively, defined by

$$\boldsymbol{\kappa} = \mathbf{L}\boldsymbol{\theta}, \quad \boldsymbol{\gamma} = \nabla w + \boldsymbol{\beta} \tag{3}$$

where $\boldsymbol{\beta} = (\theta_y \quad -\theta_x)^T$, $\nabla = (\partial/\partial x, \partial/\partial y)^T$ is the gradient vector and $\mathbf{L}$ is a differential operator matrix given by

$$\mathbf{L} = \begin{bmatrix} 0 & \partial/\partial x \\ -\partial/\partial y & 0 \\ -\partial/\partial x & \partial/\partial y \end{bmatrix} \tag{4}$$

The standard Galerkin weakform of the static equilibrium equations for the Reissner–Mindlin plate can now be written as

$$\int_\Omega \delta\boldsymbol{\kappa}^T \mathbf{D}_b \boldsymbol{\kappa}\, d\Omega + \int_\Omega \delta\boldsymbol{\gamma}^T \mathbf{D}_s \boldsymbol{\gamma}\, d\Omega - \int_\Omega \delta w p\, d\Omega = 0 \tag{5}$$

in which, the bending stiffness constitutive matrix $\mathbf{D}_b$ and the transverse shear stiffness constitutive matrix $\mathbf{D}_s$ are defined as

$$\mathbf{D}_b = \frac{Et^3}{12(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}, \quad \mathbf{D}_s = \chi t G \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{6}$$

For the free vibration analysis of Reissner–Mindlin plates, the standard Galerkin weakform can be derived from the dynamic form of energy principle

$$\int_\Omega \delta\boldsymbol{\kappa}^T \mathbf{D}_b \boldsymbol{\kappa}\, d\Omega + \int_\Omega \delta\boldsymbol{\gamma}^T \mathbf{D}_s \boldsymbol{\gamma}\, d\Omega + \int_\Omega \delta\mathbf{u}^T \mathbf{m}\ddot{\mathbf{u}}\, d\Omega = 0 \tag{7}$$

in which, $\mathbf{m}$ is the matrix containing the mass density of the material ρ and thickness t as

$$\mathbf{m} = diag[\rho t, \rho t^3/12, \rho t^3/12] \tag{8}$$

In the case of in-plane buckling analysis and assuming prebuckling stress $\boldsymbol{\sigma}^0$, nonlinear strains appear and the weak form can be written as

$$\int_\Omega \delta\boldsymbol{\kappa}^T \mathbf{D}_b \boldsymbol{\kappa}\, d\Omega + \int_\Omega \delta\boldsymbol{\gamma}^T \mathbf{D}_s \boldsymbol{\gamma}\, d\Omega + \int_\Omega \delta\boldsymbol{\varsigma}^T \boldsymbol{\tau}\boldsymbol{\varsigma}\, d\Omega = 0 \tag{9}$$

where ς and $\boldsymbol{\tau}$ are defined as

$$\boldsymbol{\varsigma} = \begin{bmatrix} w_{,x} & 0 & 0 \\ w_{,y} & 0 & 0 \\ 0 & \theta_{x,x} & 0 \\ 0 & \theta_{x,y} & 0 \\ 0 & 0 & \theta_{y,x} \\ 0 & 0 & \theta_{y,y} \end{bmatrix} \tag{10}$$