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Contact model for elastoplastic analysis of half-space indentation by a spherical impactor

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ABSTRACT

This paper presents a new contact model for analysis of post-yield indentation of a half-space target by a spherical indenter. Unlike other existing models, the elastoplastic regime of the present model was modelled using two distinct force–indentation relationships based on experimentally and theoretically established indentation characteristics of the elastoplastic regime. The constants in the model were derived from continuity conditions and indention theory. Simulations of the present model show good prediction of experimental data. Also, an approach for determining the maximum contact force and indentation of an elastoplastic half-space from the impact conditions has been proposed.

1. Introduction

Contact phenomenon finds application in many impact processes such as forging, stamping, shot-peening, and impact of projectiles on structures. Of primary interest are cases that involve plastic deformation and/or damage. Contact models that account for the effects of plastic deformation and/or damage are needed to facilitate analytical studies of post-yield impact events, to validate finite element models, and to validate indentation test results. In this study, a simple contact model that accounts for plastic deformation is formulated to model the indentation of a metallic target by a spherical indenter.

Impact is a dynamic phenomenon that involves two bodies coming in contact at a relative velocity. Generally speaking, the impact between two solid bodies gives rise to a localised deformation of the contacting bodies at the region of contact. Either both bodies are deformed locally or one of the bodies is deformed locally depending on the relative rigidity of the bodies. As a result of the occurrence of local deformation during impact between two solid bodies, the theoretical modelling of such impact events combines the contact mechanics of the localised deformation with the equations of motion of the contacting bodies. The contact mechanics for rate-independent impact response is developed based on static conditions and it gives the relationship between the force at the point of contact and the localised deformation of the contacting bodies. The sum of the local deformation of each of the contacting bodies is called the *indentation or relative approach* and this indentation is directly related to the contact force.

In several previous experimental investigations and for many practical impact problems, the conditions are such that one of the contacting bodies deforms locally and is relatively large compared to the other body. The latter is often stiff enough to withstand significant local deformation and can be considered to be completely rigid. The body that deforms locally is called the *target* and the rigid body is called the *indenter, impactor* or *projectile*. This kind of impact problem is referred to as a *half-space impact* and it is depicted in Fig. 1 for the case of elastoplastic impact. It is customary to develop contact models for rate-independent impact analysis based on half-space conditions [1]. For instance, the Hertz contact theory gives the force–indentation relationship for the impact of a spherical indenter on an elastic half-space. A detailed treatment of the Hertz contact theory for different contacting surfaces can be found in [1,2].

The Hertz contact theory is only applicable for small indentations where the impact is elastic. If the applied contact force is high enough so that the stress generated in the contact zone goes beyond the yield point, then plastic deformation occurs and significant deviations from Hertz contact theory are observed [3]. It has been shown that plastic deformation can occur at impact velocities as low as 0.14 m/s [1]. Moreover, most practical impact events are characterised by plastic deformation [1,2], and therefore it is

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necessary to account for plastic deformation in the contact model. The effect of plastic deformation is to reduce the contact force from that predicted by purely elastic deformations, and to produce a permanent indentation at the end of the contact, thereby, making the unloading path to be different from that of the loading.

Contact models that account for plastic deformation have been developed for metallic targets [1,2,5,6,8] and for composite laminate targets [4,7]. These models can be divided into two groups, namely: those that are developed based on two distinct loading regimes [2,4,7], and those that are developed based on three loading regimes [1,5,6,8]. With metallic targets, three loading regimes (elastic, elastoplastic, and fully plastic) and an unloading regime are required for complete description of the contact mechanics [1,5]. For a contact model that is based on elastic-elastoplastic-fully plastic loading regimes, the loading features and force–indentation relationships for the elastic and fully (perfectly) plastic loading regimes are well established. The elastic loading regime starts from the beginning of the loading and ends at the yield point, where the mean contact pressure is given as $P_0 = 1.1S_y$ (S_y is the yield strength of the target). The elastic loading regime can be modelled accurately using the well-known Hertz force–indentation relationship. The fully plastic loading regime is characterised by a linear force–indentation relationship and a constant mean pressure of $P_0 = 2.8S_y$, which is equal to the Brinell hardness of the target [8]. The onset of the fully plastic regime occurs at the point where the mean pressure is $2.8S_y$. The main difficulty lies in modelling the elastoplastic loading regime which starts at the yield point and ends at the onset of fully plastic conditions.

Li and co-workers [6] developed an elastoplastic contact model that is based on three loading regimes. The onset of the elastoplastic loading regime was proposed to lie at a mean pressure of $P_0 = 1.6S_y$ and this elastoplastic loading regime was modelled using an equivalent ratio power relationship. The power law has three constants that depend on the material properties of the contact system and experimental data. However, the constants were determined from FEA results for the particular case considered. Also, the restitution model used in the study does not guarantee a smooth transition from the loading to the unloading phase. A more rigorous and recent contact model that captures plastic deformation effect and is based on three loading regimes has been provided by Brake [8], who modelled the elastoplastic loading regime by enforcing continuity between the yield point and the onset of fully plastic loading using a cubic Hermite interpolation polynomial. However, it was observed that the interpolation used in the elastoplastic regime produced an unrealistic bend sometimes. To obtain a more realistic fit, the onset of the fully plastic regime was moved from the initial point to a point where the slope of the fully plastic regime is parallel to the slope of the elastic regime. A restitution model that determines the unloading path based on the loading regime from which the unloading began was also developed. The restitution model provides a smooth transition from the loading to the unloading. A drawback of Brake's model for the elastoplastic loading regime is that it requires much more computational effort compared to the other models [1,4,5,7].

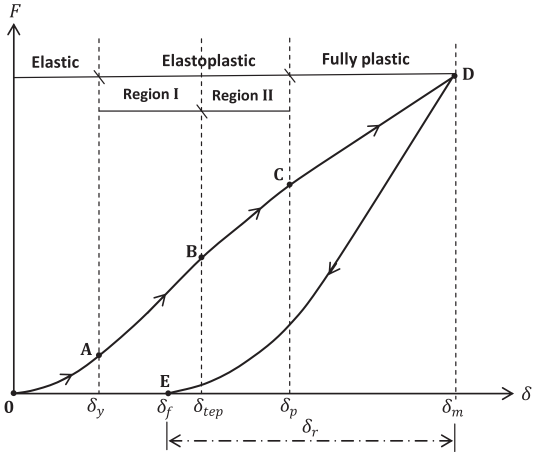

Fig. 2. Sketch of compliance curve for the new contact law.

In this paper, a new contact model that accounts for plastic deformation effects during the loading and restitution phases of the contact force is presented. The present study is motivated by (i) the gaps found in the attempts by past investigators to model the post-yield loading and restitution phases of a contact force, and (ii) the need to provide a simple and yet reasonably accurate contact model that can be easily used for impact modelling. The contact model is based on three loading regimes consisting of elastic, elastoplastic, and fully plastic regimes, and a single-phase restitution that is elastically nonlinear. Furthermore, the elastoplastic loading regime is divided into two distinct regions, and each region is modelled with a different force–indentation relationship that depicts the deformation mechanism observed from experiments and FEA results. A sketch of the compliance curve for the new contact model is shown in Fig. 2. The structure of the remainder of this paper is as follows. In Section 2, the force-indention models for the loading regimes and the unloading are presented, and the constants in these models are derived theoretically and from continuity conditions. In Section 3, the normalised form of the force–indentation models are presented. Section 4 discusses the results from the simulation of the present model in comparison with experimental data and results from other models. Finally, Section 5 gives the conclusions of the present study.

2. Model development

Static contact models can be used for modelling the impact response of structures by redefining the local indentation during impact as the relative displacement between the impactor and

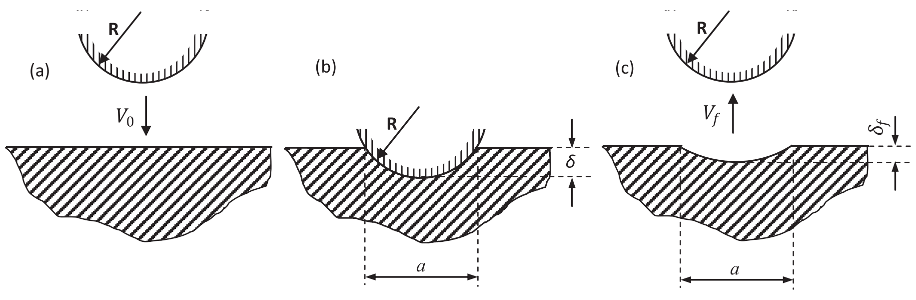

Fig. 1. Elastoplastic half-space impact of a rigid spherical indenter on a compliant flat target: (a) before impact, (b) during impact and (c) after impact.