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An adaptive quadratic approximation for structural and topology optimization

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ABSTRACT

In dual sequential methods for structural and topology optimization, approximations based on diagonal quadratic Taylor series expansion methods are frequently used to construct tractable sub-problems. However, when approximating the second-order terms, some inconsistent enforcements are usually employed to ensure the convexity of the approximations (Groenwold et al. (2010)), which may cause convergence problems in the optimization process. In this paper, an adaptive quadratic approximation (AQA) is proposed to improve robustness and convergence performance of the optimization process. Numerical results on representative structural and topology optimization problems show the efficiency of the new proposed method over other existing algorithms.

1. Introduction

Approximation schemes are typically used in structural and topology optimization problems, wherein the objective and the constraint functions are approximated using appropriate schemes at design points to construct an approximated sub-problem, which is then solved using suitable optimization methods. Thus, the original difficult problem at each optimization step is replaced by a tractable fast-to-compute approximated sub-problem. Since Schmit [2–4] first introduced the approximation concept in structural optimization in the late 1970s, extensive work has been done in the area of developing high quality approximated sub-problems. In early structural optimization studies [4–6], sub-problems were created wherein the constraints were approximated using the first-order Taylor series expansion in reciprocal variables since such approximations can appropriately capture the behavior of constraints. Fleury et.al [7] presented an approximation scheme wherein first-order Taylor series expansion in both direct and reciprocal variables are used to create a sub-problem. This method yields a sub-problem that is convex and separable and can be efficiently solved by dual methods [8,9]. This idea directly led to the convex linearization (CONLIN) scheme [10], which is demonstrated to be a successful algorithm in solving structural optimization problems. The approximations in direct and reciprocal variables are first order local approximations and are valid in a relatively small vicinity near the design point at which the approximation is carried out. Such approximations, by their construction, are not very accurate away from the expansion point. All approximations of this kind are categorized into local approximation by Barthelemy and Haftka [11].

To obtain better approximation, multi-point information can be used to construct reliable mid-range approximations that are valid in a relatively larger region near the expansion point. Haftka [12] introduced modified reciprocal approximation and two-point projection method by enforcing derivative information at current and previous design points. By linearizing the exponential intervening variable, Fadel [13] proposed a two-point exponential approximation in which the exponent is calculated by matching the approximate and exact function derivatives at two different design points. To relax the conservatism in the CONLIN method and using the previous point information, Svanberg [14] combined the modified reciprocal approximation and CONLIN to develop the Method of Moving Asymptotes (MMA). The moving asymptotes can adjust the conservatism of MMA approximations by judging the oscillation behavior of the current and previous design points, so that the approximation becomes adaptive and more efficient than CONLIN.

The above methods have been shown to be successful in lot of applications, but they are all first-order and monotonous schemes, and therefore are not suitable to approximate non-monotonous structural behaviors. Indeed, it is shown that these methods have poor convergence properties or even fail to converge for some particular problems [15,16]. The limitation of first-order approximation is also discussed by Fleury et al. [9], and in the same study a second order approximation based on Taylor series expansion is

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shown to achieve the best compromise between conservativeness and accuracy. Besides using the fully populated Hessian matrix to construct the approximation, Fleury [17] showed that approximation formulated by using only a diagonal Hessian is a more effective way that is amenable to convex programming without much accuracy loss by comparing with the sequential quadratic programming (SQP) method. Intervening variables can also be extended to second-order approximation, and the two point adaptive nonlinear approximations (TANA) schemes are of this type [18,19]. TANA schemes add second-order correction term to exponential approximation and are demonstrated to be effective for optimizing truss problems. Similarly, a globally convergent MMA (GCMMA) algorithm is presented by Svanberg [15,20] in which a second-order term is added to the MMA approximation. To adjust the second-order term, an additional inner loop is used in GCMMA which increases the computational effort as compared to MMA. Furthermore, to approximate the second-order term in GCMMA, Bruyneel and Fleury [16,21] presented a family of MMA approximations and proposed a selection scheme for different conditions. More recently based on the concept of using diagonal Hessian approximation, Groenwold et al. [22] proposed a family of diagonal quadratic approximations (DQA) based on reciprocal and exponential approximations. Also if dual methods are to be used, which are efficient for large scale topology optimization problems, DQA can always give the primal–dual relationship in a closed form, while TANA and GCMMA approximations are not amenable to dual methods as additional numerical analysis is required to construct the dual sub-problem. The detailed application of DQA on structural and topology optimization problems is provided in the recent studies by Groenwold et al. [1,23,24].

When solving large-scale structural/topology optimization problems, the computation and storage of fully populated Hessian becomes expensive; therefore, another benefit gained from using a diagonal Hessian approximation is that the algorithm is computationally efficient. Furthermore, ignoring the off-diagonal entries of Hessian makes an approximation separable, and then dual method can be easily used with such approximations [17]. Hence, the sub-problems based on the DQA approximations can be solved efficiently by dual method to get the new starting design point for the next iteration. This process continues sequentially until a convergence criteria is satisfied, so it is also called dual method based on sequential approximation (DSA) [25]. Although DQA proposed by Groenwold [22] are quite suitable for DSA framework, they are not problem-free approximations. In order to satisfy the convexity and separability requirements of DSA algorithm, some spurious enforcements in the DQA have been proposed [1]. These enforcements ensure convexity of approximation where the derivative is greater than zero by sacrificing the approximation accuracy and this drawback is illustrated in Fig. 1. Under this enforcement, when the gradient at a current design point x_i^k is positive (where k is the iteration number), the reciprocal or exponential approximation becomes concave so the second derivative information they yield becomes negative and, therefore the direct approximation without any enforcement is concave. In this case, the enforced convex approximation is constructed by changing the sign of derivatives [1]; however, this is inaccurate and the enforced approximation cannot represent the true behavior of the actual function anymore, and the optimal point of the enforced approximation x_i^{k+1} is different from the actual optimal value x_i^*, as shown in Fig. 1. Thus, although DQA family proposed by Groenwold [22] yields non-monotonous approximations, the approximation may not be accurate under certain conditions, and this may eventually result in slow convergence. Alternatively, if these enforcements are relaxed or improved by an adaptive approximation, the approximation accuracy will increase and this will finally speed up the convergence.

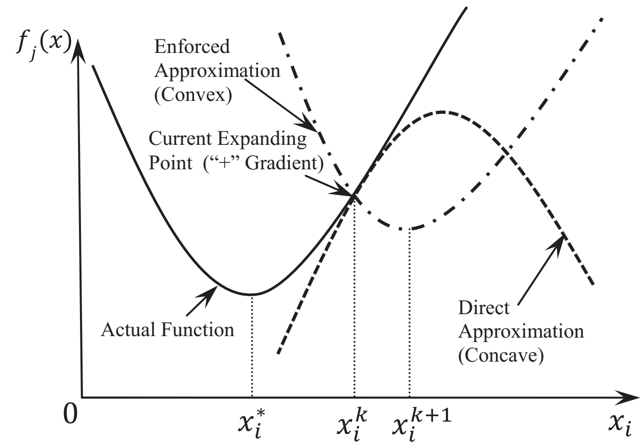

Fig. 1. Illustration of enforcement for positive gradient.

In this paper, an adaptive quadratic approximation (AQA) approach is proposed based on the DQA to overcome the difficulties mentioned above by selecting proper approximations adaptively instead of direct enforcements. The AQA approximation is shown to yield better results for structural and topology optimization problems than the enforced DQA methods. The representative test problems included in this paper are: truss optimization problems, Fleury's weight like optimization problem, and topology optimization for minimum compliance and compliant mechanisms design.

The paper is arranged as follows: In Section 2, the outline of DSA algorithm is provided. Then the various monotonous and non-monotonous approximations together with their advantages and restrictions are discussed in Section 3. In order to address the restrictions described in Section 3, a new adaptive quadratic approximation (AQA) scheme is proposed in Section 4. Performance evaluation of the new proposed algorithm is carried out by the comparison of various algorithms via numerical experiments in Section 5. Finally, the important concluding remarks are offered in Section 6.

2. Dual sequential approximation (DSA) algorithm

2.1. Nonlinear optimization problem

The primal structural/topology optimization problem can be described as:

$$\begin{aligned} &\min_{\boldsymbol{x}\in\mathcal{D}} \quad f_0(\boldsymbol{x}) \\ &\text{Subject to}: \\ &f_j(x) \leqslant 0, \quad j = 1,2,\ldots,m \\ &\mathcal{D} = \{x_i | x_{li} \leqslant x_i \leqslant x_{ui}\}, \quad i = 1,2,\ldots,n \end{aligned} \tag{1}$$

where $\boldsymbol{x} \in \mathcal{D} \subset \mathbb{R}^n$ is the design variable; $f_0(\boldsymbol{x})$ is the objective function which to be minimized; $f_j(\boldsymbol{x})$ are the inequality constraints, and x_{li} and x_{ui} are the upper and lower boundaries of the i^{th} design variable x_i, respectively. In most cases, $f_0(\boldsymbol{x})$ and $f_j(\boldsymbol{x})$ represent the linear or nonlinear responses of a structure, typically total weight, displacement at certain nodes, and stiffness or compliance of a structural system, and are assumed to be at least once continuously differentiable. In structural optimization problems, the design variables typically represent area of cross section or any other geometric property of structural members while in topology optimization