ARTICLE IN PRESS

Computers and Structures xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

An efficient two-node finite element formulation of multi-damaged beams including shear deformation and rotatory inertia

Marco Donà [a], Alessandro Palmeri [a,*], Mariateresa Lombardo [a], Alice Cicirello [b]

[a] School of Civil and Building Engineering, Loughborough University, Sir Frank Gibb Building, Loughborough LE11 3TU, England, United Kingdom
[b] Department of Engineering, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, England, United Kingdom

ARTICLE INFO

Article history:
Accepted 1 October 2014
Available online xxxx

Keywords:
Cracked beams
Euler–Bernoulli beam theory
Damaged structures
Finite element analysis
Timoshenko beam theory
Rotatory inertia

ABSTRACT

A computationally efficient beam finite element is presented for the static and dynamic analysis of frame structures with any number and location of concentrated damages, whose macroscopic effects are simulated with a set of longitudinal, rotational and transversal elastic springs at the position of each singularity. The proposed mathematical model exploits positive Dirac's deltas in the corresponding flexibility functions of the beam elements, and allows also considering shear deformations and rotatory inertia. Such contributions may have a huge impact on the higher modes of vibration, as confirmed by the numerical examples.

1. Introduction

Presence of damages in frame structures may substantially change their static and dynamic response, reducing the performance and eventually leading to failure. Broadly speaking, the approaches available in the literature to model cracks and other forms of concentrated damage in beams and columns can be classified into three main categories: local stiffness reduction (LSR), discrete spring (DS) equivalent models, and more sophisticated formulations adopting methods and concepts of Fracture Mechanics [1].

The LSR is conceptually the simplest approach to build a finite element (FE) model of a damaged beam, as it just requires to mesh the member with a sufficient number of beam elements and to reduce the relevant stiffness component (e.g. the flexural stiffness) of the element at the position where the damage occurs (see Fig. 1(a)) [2–4]. To be efficient, this approach requires a fine mesh, and the problem arises of quantifying the stiffness reduction in each FE to match the global effects of the actual concentrated damage.

In the DS model, on the contrary, the beam is ideally divided at the position of the damage into two regions; the new elements so obtained are articulated at this location and, depending on the characteristics of the damage, the residual stiffness is simulated by axial, and/or rotational and/or shear springs. For slender beams in bending, for instance, the presence of cracks affects mainly the flexural stiffness *EI*, and therefore the DS model just consists of joining the adjacent split elements of the beam with a hinge (i.e. axial and shear flexibility are not considered), which are then coupled with a rotational spring, whose stiffness is related to the intensity of the damage (see Fig. 1(b)): that is, the severer the damage, the softer is the spring. In the limiting case in which the cross section is fully damaged, the stiffness of the spring becomes zero. The main shortcoming with the DS model is that, if a conventional beam element is used, two additional FE nodes must be placed at the location of each concentrated damage, i.e. one node on each side. This could be particularly cumbersome if the DS model is used for the purposes of damage identification, as it would also require re-meshing the beam during the identification process.

Alternatively, the use of 2D or 3D FE models may produce very detailed and accurate results, but such computationally intensive approaches are more appropriate to tackle problems of crack initiation and/or propagation, while global analysis of framed structures and damage detection in beams and columns can be carried out by less sophisticated FE models. As a matter of fact, the DS model often provides the best trade-off between accuracy and computational effort for these applications.

Motivated by these considerations, many formulations have been developed for the DS model, including the "rigidity modelling" by Biondi and Caddemi [5,6], in which the singularities in the flexural stiffness, corresponding to concentrated damages, are introduced as negative impulses (i.e. Dirac's delta functions with

* Corresponding author.
E-mail addresses: a.palmeri@lboro.ac.uk, dynamics.structures@gmail.com (A. Palmeri).

http://dx.doi.org/10.1016/j.compstruc.2014.10.002
0045-7949/

negative sign). Although expedient, this mathematical representation is not consistent with the definite-positive nature of the flexural stiffness, delivering however exact closed-form solutions for the static analysis of multi-cracked slender beams in bending. Aimed at overcoming this theoretical flaw, Palmeri and Cicirello [7] have recently presented a (physically consistent) dual representation of cracks, i.e. a "flexibility modelling", in which Dirac's delta functions with a positive sign are introduced in the bending flexibility of the beam, i.e. the inverse of its flexural stiffness; they also extended the model to cope with Timoshenko beams, to take into account the contribution of the shear deformations in the uncracked regions of the member. A rigorous theoretical justification of this flexibility modelling has been subsequently presented in Ref. [8].

The DS model has been extensively studied and applied within different schemes of structural health monitoring, aimed at identifying presence, location and severity of concentrated damage in frame structures (e.g. Refs. [9–13]), and the papers cited in the literature review of [14]. In this context, the size of the FE assembly for the structural frame under investigation plays an important role, as ideally it should be as small as possible. Indeed, the vast majority of the identification algorithms proceed iteratively until convergence, and therefore any little saving in the computational cost for a single analysis may result in a significant advantage on the whole process. Moreover traditional damage detection approaches may require FE re-meshing throughout the identification process, which inevitably causes an additional increase in the computational effort.

Aimed at addressing these issues, an analytical and numerical study has been carried out to develop an efficient two-node multi-damaged beam (MDB) element for the FE analysis of frame structures, which is able to account for any number and location of concentrated damages without increasing the size of the problem in comparison with the corresponding undamaged structure. It is worth mentioning here that within the present study, any localised increase in the flexibility of the beam is considered as a concentrated damage, provided that the portion of the beam affected by such increase is less than its cross sectional dimensions.

Similar approaches have been recently pursued by other authors, whose studies differ in the analytical formulations adopted to get the closed-form expressions for stiffness matrix, load vector and mass matrix. In the FE proposed by Skrinar [15,16], cubic splines are used to represent the field of transverse displacements in each uncracked region of the beam, while the additional kinematic and static unknowns arising at each crack have been eliminated with the help of compatibility and equilibrium equations combined with the Hooke's law for the rotational springs simulating the cracks. However, this study has only considered slender Euler–Bernoulli beams in bending and masses lumped at the two nodes of the resulting FE, which may limit its applicability.

The formulation proposed by Caddemi et al. [17] is more general, as it includes the shear deformations (i.e. the Timoshenko beam theory has been adopted), and rotational and transverse springs are considered at the position of each crack. They have employed the rigidity modelling of concentrated damage to derive the exact closed-form expressions for the deformed shape of the two-node multi-cracked beam element subjected to unitary nodal settlements, which in turn have been used to derive the stiffness matrix and consistent mass matrix. Differently from this formulation, the proposed work includes the axial damage in the MDB element, so that it is possible to consider for each concentrated damage a set of axial, rotational and shear springs, i.e. the beam is fully articulated at the position where the concentrated damage occurs, allowing relative longitudinal, rotational and transverse movements (this is particularly important in the dynamic analysis of structures in which both axial and transversal deformations affects the modes of vibrations, as for instance in camshaft [18,19]. The proposed formulation can also be useful when a concentrated increase in the flexibility of the structural member comes from an internal joint, e.g. a beam-to-beam connection in a steel frame, rather than from a proper damage.

This paper extends the preliminary work by Donà et al. [20] with the inclusion of the rotatory inertia, which results in lowering the vibrational frequencies, particularly the higher ones, and therefore can be critical in the accurate identification of position and severity of the damages [21,22]. Furthermore, when the material has a relatively high ratio of bending to shear modulus, e.g. wood, the effects of shear deformations and rotatory inertia may become negligible even for slender beam [23]. In order to facilitate the practical implementation of the proposed MDB element, the closed-form expressions for stiffness matrix (see Appendix A) and consistent mass matrix (see Appendix B) are also provided.

The paper clearly demonstrates the improved efficiency of the proposed two-node MDB element with respect to the LSR model as well as the effects of the inclusion of both shear deformations and rotatory inertia. To do this, the results of static analyses are validated against those provided by the commercial FE code SAP2000 [24]. It is shown that, independently of the number of concentrated damages, the proposed MDB element is able to deliver, for both Euler–Bernoulli and Timoshenko kinematic models, the same exact solutions with just a single FE for each beam and column in the frame structure, while the LSR model only gives an approximate solution (whose accuracy depends on the size of the mesh) and SAP2000 needs an additional node at the position of each damage. Both lumped and consistent mass matrices have been also tested for the modal analyses. It is shown that using lumped masses with the proposed MDB element allows recovering the same eigenproperties given by SAP2000, provided that the same mesh is adopted; besides, using the consistent mass matrix increases the accuracy, as the eigenproperties so obtained converge more rapidly to the exact solution, with the additional advantage that the FE mesh is independent of the position of the concentrated damages.

2. Exact closed-form solutions for beams with multiple concentrated damages under axial and transverse loads

Aimed at defining the shape functions for the proposed MDB element, the flexibility modelling recently proposed by Palmeri and Cicirello [7] for a concentrated flexural damage (i.e. a crack-induced lumped rotation) has been extended to include axial and shear lumped deformations at the position of the concentrated damage, and has been then used to derive the exact closed-form solutions for MDBs subjected to both axial and transverse loads.

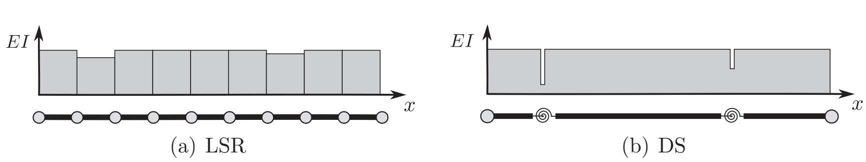

Fig. 1. The bending stiffness EI for the LSR (a) and DS (b) models.