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T-spline based XIGA for fracture analysis of orthotropic media

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ABSTRACT

Fracture analysis of orthotropic cracked media is investigated by applying the recently developed extended isogeometric analysis (XIGA) (Ghorashi et al., 2012) using the T-spline basis functions. The signed distance function and orthotropic crack tip enrichment functions are adopted for extrinsically enriching the conventional isogeometric analysis approximation for representation of strong discontinuity and reproducing the stress singular field around a crack tip, respectively. Moreover, by applying the T-spline basis functions, XIGA is further developed to make the local refinement feasible. For increasing the integration accuracy, the 'sub-triangle' and 'almost polar' techniques are adopted for the cut and crack tip elements, respectively. The interaction integral technique developed by Kim and Paulino (2003) is applied for computing the mixed mode stress intensity factors (SIFs). Finally, the proposed approach is applied for analysis of some cracked orthotropic problems and the mixed mode SIFs are compared with those of other methods available in the literature.

1. Introduction

Orthotropic materials such as composites have been increasingly applied in many engineering applications e.g. aerospace, automobile and marine structures because of their high strength and stiffness to weight ratios. Considering their strength, they are applied in thin shell forms with the possibility of crack initiation and propagation. As a result, fracture analysis of such media has been the center of attention for many researchers in the past few decades.

Analytical solution of stress and displacement fields for an orthotropic plate with a crack has already been obtained by Sih et al. [3]. Some other analytical investigations on fracture behavior of composites can be found in [4–10]. As the analytical methods are not feasible in resolving practical engineering problems, numerical methods are better alternatives.

The remeshing requirement and the existence of singular fields around crack tips in simulation of crack propagation problems led to the development of several computational approaches such as meshfree methods [11–21] and the extended finite element method (XFEM) [22–24]. Problems involving with moving discontinuities such as crack propagation can be analyzed by these methods without the requirement of remeshing or rearranging of the nodal points. Some applications of these methods can be found in [25–31]. In XFEM, a priori knowledge of the solution is locally added to the approximation space. This enrichment allows for accurate capture of particular features such as discontinuities and singularities which are present in the solution.

In order to analyze the problem of cracked orthotropic bodies different approaches have been applied such as the hybrid-displacement finite element method [32], the boundary element method (BEM) [33], finite elements and the modified crack closure method [34]. Asadpoure et al. [35–37] succeeded in developing three sets of orthotropic enrichment functions for different types of composites using the analytical solutions and implemented them within an XFEM framework. The general form of orthotropic enrichment functions [37] have also been adopted in the enriched element free Galerkin (EFG) method [38]. Further developments have been reported for dynamics and moving cracks in orthotropic media [39,40] and delamination analysis of composites [41].

Although cracked orthotropic media have been studied by several different methods, the new developments in the promising computational approach of the extended isogeometric analysis (XIGA) [42,1] have attracted new investigations to further apply it for such problems.

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XIGA takes the advantages of its two origins: the extended finite element method (XFEM) and the isogeometric analysis (IGA) [43]. IGA integrates the computer aided design (CAD) into the finite element method (FEM) using the concept of isoparametric elements, in which the same shape functions are used to represent the geometry and to approximate the solution. Its superiorities over conventional FEM are: capability of exact representation of complex geometries regardless of the mesh coarseness, simplification of the refinement process and improvement of the solution accuracy. A large variety of problems [44–46] have already been solved by IGA. XIGA has been successfully applied for simulation of stationary and propagating cracks in 2D linear-elastic isotropic media [1]. Ghorashi et al. have further enhanced XIGA for analysis of curved cracks [47]. In addition, more results have been reported on crack detection using the XIGA [48]. Recently, the XIGA has been also applied for analysis of material interface problems and the techniques for achieving optimal convergence rates have been addressed [49].

The current XIGA method adopts the conventional NURBS basis function which has a considerable drawback. The local refinement cannot be defined within it because it is based on a tensor-product structure which requires the control points to lie topologically in a rectangular grid. In other words, when a control point needs to be added, several superfluous control points should be defined. One solution is to use multiple patches, which has also some limitations. The compatibility of adjacent NURBS patches on their interfaces has to be maintained. Generally, the refinement extends from one patch to another unless the compatibility between patches are enforced in a different way. One can weakly enforce it by the variational formulation applying the discontinuous Galerkin formulation. Another alternative is to enforce C^0 continuity on the interfaces between patches [50] by utilizing constraint equations for the control points and variables. IGA formulations based on triangular splines [51] might be an interesting alternative as they simplify the mesh generation and adaptive refinement procedure.

A more enchanting solution for the aforementioned problem is to use the so-called T-splines [52–54], which are a generalization of NURBS, by allowing a row of control points to terminate before reaching the patch boundary. This feature enables the truly local refinement without extending the entire row of control points. Furthermore, by using the T-splines several NURBS patches that have different knot vectors can be efficiently merged into a single gap-free model of C^0 or higher order continuity [53]. Recently, a subset class of T-splines called "analysis-suitable T-splines", which are linearly independent, has been introduced in the IGA framework [55] and a highly localized refinement algorithm which meets the demands of both design and analysis has been presented [56].

Another alternative to T-splines are PHT-splines or RHT-splines that are based on hierarchical T-meshes [57]. Continuum and structural element formulations based on PHT- and RHT-splines have been developed in [58–60], respectively.

In this contribution, XIGA is further extended and the T-spline basis functions which belong to the analysis-suitable T-splines are adopted to make the local refinement for feasible adaptive procedure. Furthermore, based upon the work of Ghorashi et al. [61], the orthotropic enrichment functions [37] are adopted to investigate cracked orthotropic bodies.

In Section 2, important formulations of orthotropic materials are introduced. Basis functions including NURBS and T-splines are then described in Section 3. Thereafter, the proposed XIGA method including orthotropic enrichment functions is presented in Section 4. The results obtained from the present orthotropic XIGA and those available in the literature are compared in Section 5 to demonstrate the accuracy and efficiency of the proposed approach. This is achieved by implementing the interaction integral technique, developed by Kim and Paulino [2], for computation of mixed mode stress intensity factors. This section is followed by some concluding remarks in Section 6.

2. Fracture mechanics in orthotropic media

The stress–strain law in an arbitrary linear elastic material can be written as

$$\boldsymbol{\varepsilon} = \boldsymbol{c}\boldsymbol{\sigma} \tag{1}$$

where $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ are the strain and stress vectors, respectively, and $\mathbf{c}$ is the compliance matrix,

$$\mathbf{c}^{3D} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \tag{2}$$

where E, ν and G are Young's modulus, Poisson's ratio and shear modulus, respectively. For a plane stress case, the compliance matrix is reduced to the following form:

$$\mathbf{c}^{2D} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0 & 0 & 0 & 0 \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \tag{3}$$

and for a plane strain state,

$$c_{ij}^{2D} = c_{ij}^{3D} - \frac{c_{i3}^{3D}.c_{j3}^{3D}}{c_{33}^{3D}} \quad \text{for } i,j = 1,2,6 \tag{4}$$

Now assume an anisotropic body is subjected to arbitrary forces with general boundary conditions and a crack. The global Cartesian coordinate (X_1, X_2), the local Cartesian coordinate (x, y) and the local polar coordinate (r, θ), defined on the crack tip, are illustrated in Fig. 1. A fourth-order partial differential equation with the following characteristic equation can be obtained from the equilibrium and compatibility conditions [5],

$$c_{11}s^4 - 2c_{16}s^3 + (2c_{12} + c_{66})s^2 - 2c_{26}s + c_{22}0 = 0 \tag{5}$$

where $c_{ij} \quad (i,j = 1,2,6)$ are the components of $\mathbf{c}^{2D}$. According to [5], the roots of Eq. (5) are always complex or purely imaginary $(s_k = s_{kx} + is_{ky}, \quad k = 1,2)$ and occur in conjugate pairs as $s_1, \bar{s_1}$

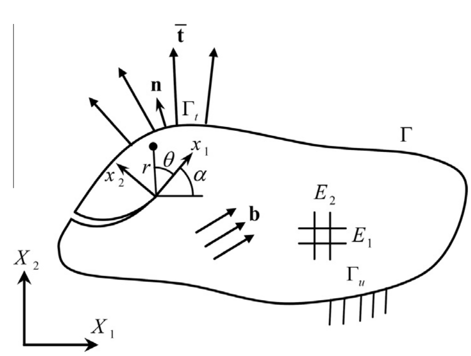

Fig. 1. An arbitrary orthotropic cracked body subjected to body force **b** and traction **t̄**.

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