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Post-buckling analysis of rectangular plates comprising Functionally Graded Strips in thermal environments

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ABSTRACT

Description is given of a semi-analytical finite strip method for analysing the post-buckling behaviour of functionally graded rectangular plates in thermal environments where plates are under uniform, tent-like or nonlinear temperature change across the thickness. The material properties are assumed to vary through the thickness according to the power law. The formulations are based on the classical plate theory and the concept of the principle of the minimum potential energy. The Newton–Raphson method is used to solve the equilibrium equations. A range of applications are described and the numerical results are compared to the available results, wherever possible.

1. Introduction

Functionally graded materials (FGMs) are those in which the volume fractions of two or more constituents are varied continuously as a function of position along certain dimension of the plate to achieve a required function. By gradually varying the volume fraction of constituent materials, their mechanical properties exhibit a smooth and continuous change from one surface to the other one. The ceramic constituent of the material provides the high-temperature resistance due to its low thermal conductivity. Thus FGMs have received considerable attention as one of advanced inhomogeneous composite materials in many engineering applications by eliminating interface problems and alleviating thermal stress concentrations. Most of the researches on FGMs have been restricted to thermal stress analysis, fracture mechanics, and optimization. However, very little work has been done to consider the buckling, post buckling and vibration behaviour of structures constructed of FGM.

The Finite Strip Method (FSM) is a special form of the Finite Element Method (FEM). In terms of computational expenses, not only it takes much shorter computing time and smaller amount of core for solution of comparable accuracy, but also it require very small amount of input data due to the small number of mesh lines involved. However, like any other methods, the conventional/semi-analytical FSM has its own limitations and drawbacks such as boundary conditions limitations induced by shape functions, lacking ability to analyse plates with cut-out and using only rectangular elements. These issues could be overcome by incorporating other versions of FSM, i.e. B-spline FSM and complex FSM, to name a few.

The first two authors of the current paper and their co-workers have made several contributions by developing different versions of finite strip methods, namely full-energy semi-analytical FSM [1,2], full-energy spline FSM [1], semi-energy FSM [2] and exact FSM [3].

Ge et al. investigated post buckling behaviour of composite laminated plates with the aid of the B-spline finite strip method under the combination of temperature load and applied uniaxial mechanical stress [4]. Liew et al. examined the post-buckling behaviour of functionally graded material FGM rectangular plates that are integrated with surface-bonded piezoelectric actuators and are subjected to the combined action of uniform temperature change, in-plane forces, and constant applied actuator voltage [5]. Sohn et al. worked on static and dynamic stabilities of functionally graded panels based on the first-order shear deformation theory which are subjected to combined thermal and aerodynamic loads. They derived equations of motion by the principle of virtual work and numerical solutions were obtained by a finite element method. In addition, they utilized the Newton–Raphson method to get solutions of the nonlinear governing equations [6]. The authors of the current work developed the semi-analytical finite strip method (S-a FSM) for analyzing the buckling behaviour of rectangular FGM plates under thermal loadings by incorporating the total potential energy minimization and solving the corresponding eigenvalue problem [7]. They also extended the same method to predict post-buckling behaviour of simply-supported FG plates

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subjected to the three types of thermal loadings, i.e. uniform temperature rise, tent-like temperature distribution and nonlinear temperature change across the thickness [8]. Material properties were assumed to be graded in the thickness direction according to a simple power law distribution in terms of the volume fractions of the constituents. For simplification, the properties were assumed to be temperature-independent.

It is noted that the current paper is an updated and revised version of the conference paper [8]. It is also noted that in the aforementioned conference paper and for that matter in the current paper, it is the first time that the concept of FGS is introduced for thermal post-buckling analysis by comparing two different approaches for solving heat conduction equation. The application and scope of the current paper are strengthened by studying more numerical results corresponding to different boundary conditions and approaches for solving the heat conduction equation.

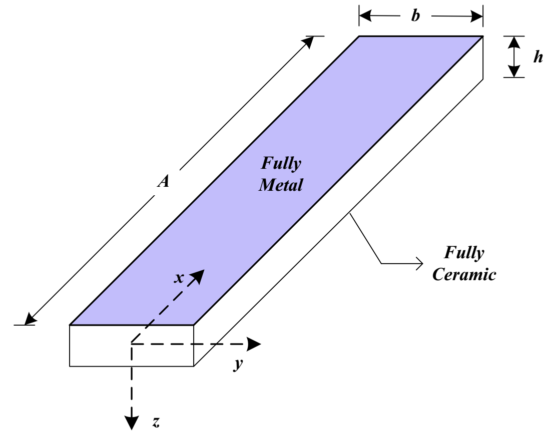

Fig. 2. A typical FGS.

2. Theoretical developments

Thin plates/shells are generally referred to those structures whose thickness is very small compared with the other two dimensions. Even though there is no precise definition for a thin plate, if the ratio of the thickness to the shorter length of other two dimensions is less than 0.05, the plate can practically be considered thin. Classical Plate Theory (CPT) allows studying thin plates once the Kirchhoff hypothesis holds:

- Straight lines perpendicular to the mid-surface before deformation remain straight after deformation.
- The transverse normals are inextensible.
- The transverse normals rotate such that they remain perpendicular to the mid-surface after deformation.

Otherwise, for analysis of thick and moderately thick plate structures, other suitable theories, i.e. higher- and first-order shear deformation theory should be taken into account by relaxing the first and the third assumptions, respectively. Thus, throughout the theoretical developments of this paper which focus on thin plate structures, an initially flat plate based on the classical plate theory is assumed. It is worth mentioning that in this plate theory, all three transverse strain components and subsequently shear stress components are zero by definition, whereas in other two developed theories, approximate distribution of shear stresses through the plate thickness is considered.

A rectangular FGP whose longitudinal and transverse dimensions are A and B, respectively, is supposed. This plate can be divided into several strips called Functionally Graded Strips (FGS) while, as illustrated in Fig. 1, they are laid parallel to one another and to the longitudinal edges of the plate.

It is noted that a single FGS (shown in Fig. 2) forms part of a rectangular FGP of length A (i.e. the same length as that of a strip) and width b (with $B \geqslant b$). The composition is assumed to vary in

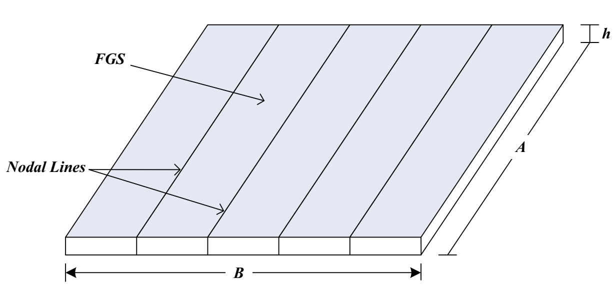

Fig. 1. Discretization of a rectangular FGP.

such a way that the upper surface of the strip is completely metal (designated by surface m at $z = -h/2$), whereas the lower surface is fully ceramic (designated by surface c at $z = h/2$). Thus, it is assumed that the material properties of the FGS such as the modulus of elasticity E, shear modulus G, thermal expansion coefficient α and thermal conduction coefficient K are changed in the thickness direction z by a function $\vartheta(z)$ which is introduced by power law variations of material properties distribution as

$$\vartheta(z) = \vartheta_{cm}\left(\frac{z}{h}+\frac{1}{2}\right)^n + \vartheta_m \tag{1}$$

while Poisson's ratio υ is assumed to be constant. In Eq. (1), ϑ_m and ϑ_c denote values of the variables at surface m and surface c of the strip, respectively and $\vartheta_{cm} = \vartheta_c - \vartheta_m$.

Moreover, in the above equation, the term $(z/h + 1/2)^n$ is known as the volume fraction of the ceramic phase, and n (volume fraction index) is non-negative real value indicating the material variation profile through the thickness direction.

As a result of the CPT assumption, the Kirchhoff normalcy condition is incorporated, and thus:

$$\begin{aligned} \bar{u}(x,y,z) &= u(x,y) - z\tfrac{\partial w(x,y)}{\partial x} \\ \bar{\upsilon}(x,y,z) &= \upsilon(x,y) - z\tfrac{\partial w(x,y)}{\partial y} \\ \bar{w}(x,y,z) &= w(x,y) \end{aligned} \tag{2}$$

where $\bar{u}, \bar{\upsilon}$ and $\bar{w}$ are components of displacement at a general point, whilst u, υ and w are similar components at the middle surfaces ($z = 0$).

Using Eq. (2) in the Green's expressions for the in-plane nonlinear strains and neglecting lower-order terms in a manner consistent with the usual Von-Karman assumptions incorporated with the thermal effects gives the following expressions for strains at a general point:

$$\varepsilon_m = \bar{\varepsilon} - \varepsilon_T = (\varepsilon_l + \varepsilon_{nl} + z\psi) - \varepsilon_T \tag{3-a}$$

where ε_m and ε_T are mechanical and thermal strains, respectively, and

$$\varepsilon_l = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial \upsilon}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial \upsilon}{\partial x} \end{Bmatrix}, \quad \varepsilon_{nl} = \begin{Bmatrix} \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \end{Bmatrix}, \quad \psi = \begin{Bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ -2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

$$\varepsilon_T = \alpha(z)\cdot\Delta T(x,y,z)\begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \tag{3-b}$$