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Reduction methods for the dynamic analysis of substructure models of lightweight building structures

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ABSTRACT

In the present study, different model order reduction methods were compared in terms of their effects on the dynamic characteristics of individual building components. A wide variety of methods were employed in two numerical examples, both being models of wooden floor structures, in order to draw conclusions regarding their relative efficiency when applied to models of such structures. It was observed that a comparison of the methods requires the reduced models to be exposed to realistic boundary conditions, free–free eigenvalue analyses being insufficient for evaluating the accuracy of the reduced models when employed in an assembly of substructures.

1. Introduction

Lightweight buildings are often constructed using prefabricated planar or volume elements, often with use of low-stiffness panels mounted on high-stiffness beams. Accurately assessing the dynamic behaviour of these elements when rather high vibration frequencies are involved requires use of finite element (FE) models representing the geometry in considerable detail. Assembling the individual elements of multi-storey lightweight buildings within the framework of global FE models of entire buildings results in very large models, the number of degrees of freedom (dofs) of which easily exceeds the limits of computer capacity, at least for computations to be performed within reasonable lengths of time. The question arises then of how such FE models can be reduced in size while at the same time being able to represent the dynamic characteristics of the building or buildings in question with sufficient accuracy. The method of dividing a large model into components and creating a global model through coupling models of reduced size of each component is referred to as substructuring. In the present study, low-frequency vibrations in multi-storey lightweight buildings are modelled by adopting a substructuring approach.

In recent decades, a number of methods for model order reduction of dynamic problems have been developed within the area of structural mechanics, mode-based methods being the methods most frequently used. Fairly recently, methods originating from control theory, designated here as modern reduction methods, have been employed within structural mechanics. In contrast to mode-based methods which have an explicit physical interpretation, the modern reduction methods are developed from a purely mathematical point of view. Some mode-based methods are implemented in commercial FE software which enables them to be applied to large-scale problems directly. In order to apply other methods to models created in commercial FE software, the system matrices involved need to be exported from the software and be reduced in another environment.

A number of comparative studies have been published in which the performance of different reduction methods has been evaluated, in connection with mechanical engineering problems. In [1,2], modern reduction methods were compared with mode-based methods. In [1], a rack consisting of steel beams was used as a numerical example, the reduction methods involved being compared by studying the structural response within the time domain and the Frobenius norm of the transfer function matrix for different load cases. It was concluded that the modern reduction methods produce excellent reduction results and are more effective than mode-based methods are. In [2], a crankshaft of a piston served as a numerical example, the Frobenius norm of the transfer function matrix being used to compare the reduction methods in question. It was concluded that substantial benefits can be achieved by use of the modern reduction methods. In [3], a wide range of methods was compared by studying the eigenfrequencies and eigenmodes of an elastic rod. The modern reduction methods were found to perform better for mechanical problems than several of the classic methods. In [4], however, in which a clamped

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beam structure served as a numerical example, it was concluded that mode-based methods are better suited for the analysis of multibody systems than modern reduction methods are. The eigenfrequencies and eigenmodes were analysed with different boundary conditions applied at the interface of the reduced models. It was concluded that mode-based methods are less dependent than the modern reduction methods are on variations in the boundary conditions, something which would clearly be an important advantage in multibody dynamics.

In the comparative studies just referred to, conclusions were drawn on the basis of numerical examples involving relatively simple structures. Lightweight floor and wall structures, however, generally have a much more complex geometry, making it difficult to extrapolate the conclusions in question. Also, in the comparative studies referred to, different types of analyses were used for evaluating the performance of the reduction methods employed, this providing diverse information that can be evaluated in a variety of ways. By applying analyses of multiple types to a given numerical example it should be possible to obtain a broader understanding of the behaviour of different reduction methods than a single type of analysis would provide. Moreover, analysing the reduced models with realistic boundary conditions is necessary since the boundary conditions employed can have a strong influence on the performance of different reduction methods, as demonstrated in [4].

The objective of the analyses carried out in the present investigation was to evaluate the performance of a rather wide range of model order reduction methods by comparing their accuracy and computational cost when applied to detailed FE models of floor and wall structures. The conclusions will be of value in the process of constructing efficient substructure models for vibration analysis of multi-storey lightweight buildings. The reduced models employed are in this paper evaluated in terms of eigenfrequencies and eigenmodes in a free-free state, as well as in terms of vibration transmission behaviour when the structures in question are exposed to realistic boundary conditions, obtained by connecting them with other building components. New insight is offered regarding both the efficiency of the reduction methods when employed in the analysis of complex structures and the effect of applying realistic boundary conditions to the reduced models.

Commercial FE software of different kinds represent convenient tools for both pre- and post-processing, such as in the coupling of substructures and in the visualisation of results. Since some reduction methods reported on in the literature are incompatible with such software, methods of this sort are either excluded from the analyses here or are used in a modified fashion. A broad range of model order reduction methods presented in the literature will be discussed and the theories behind them taken up. The performance of the reduction methods, applied to lightweight building structures, was evaluated for frequencies of less than 100 Hz by studying two numerical examples. The first example is a model of moderate size of a wooden floor structure, a model created in the commercial FE software Abaqus, from which the system matrices were exported to Matlab, in which various of the reduction methods described in Section 2 were employed. The second example is a large and detailed model of an experimental wooden floor structure, analysed with use of model order reduction methods implemented in Abaqus as well as by use of an alternative approach employing structural elements. Although the conclusions presented in this paper are based in principle on the results of the two numerical examples, many wooden floor and wall structures have geometries and materials similar to those of the structures studied in the two examples. Accordingly, the main conclusions arrived at would appear to be applicable to a wide variety of wooden floor and wall structures similar in topology to these two floors.

2. Model order reduction

An FE formulation of a structural dynamics problem results in a linear equation of motion of the following form [5]:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F}, \tag{1}$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K} \in \mathbb{R}^{n\times n}$ are the mass, damping and stiffness matrices respectively, $\mathbf{F} = \mathbf{F}(t) \in \mathbb{R}^{n\times 1}$ is the load vector and $\mathbf{u} = \mathbf{u}(t) \in \mathbb{R}^{n\times 1}$ is the state vector which is sought. A dot denotes differentiation with respect to time, t. The objective of model reduction here is to find a system of m dofs in which $m \ll n$, one which preserves the dynamic characteristics of the full model. The general approach is to approximate the state vector by use of the transformation $\mathbf{u} = \mathbf{T}\mathbf{u}_R$, where $\mathbf{T} \in \mathbb{R}^{n\times m}$ is a transformation matrix and $\mathbf{u}_R \in \mathbb{R}^{m\times 1}$ is a reduced state vector. Applying the transformation in question to Eq. (1) results in

$$\mathbf{M}_R\ddot{\mathbf{u}}_R + \mathbf{C}_R\dot{\mathbf{u}}_R + \mathbf{K}_R\mathbf{u}_R = \mathbf{F}_R, \tag{2}$$

$$\mathbf{M}_R = \mathbf{T}^T\mathbf{M}\mathbf{T}, \quad \mathbf{C}_R = \mathbf{T}^T\mathbf{C}\mathbf{T}, \quad \mathbf{K}_R = \mathbf{T}^T\mathbf{K}\mathbf{T}, \quad \mathbf{F}_R = \mathbf{T}^T\mathbf{F}, \tag{3}$$

where $\mathbf{M}_R, \mathbf{K}_R, \mathbf{C}_R \in \mathbb{R}^{m\times m}$ are the reduced mass, damping and stiffness matrices, respectively, and $\mathbf{F}_R \in \mathbb{R}^{m\times 1}$ is the reduced load vector. In recent decades, many different methods for model order reduction, involving procedures of varying types for establishing the transformation matrix and the reduced state vector involved, have been proposed in the literature. The dofs in the reduced state vector can be divided into two categories: physical dofs and generalised coordinates. Physical dofs are the dofs of the full system that are retained in the reduction process, whereas the generalised coordinates represent the amplitudes of various Ritz basis vectors [6] that describe the deflection shapes that are allowed in the reduced system. The reduction methods can be categorised according to the type of dofs generated in the reduction process, where *condensation methods* involve only physical dofs, *generalised coordinate methods* are based solely on generalised coordinates, and *hybrid reduction methods* employ a combination of dofs of both types. A number of important methods within each category are listed below:

- Condensation methods
  - Guyan reduction [7]
  - Dynamic reduction [8]
  - Improved reduction system (IRS) [9,10]
  - System equivalent expansion reduction process (SEREP) [11]
- Generalised coordinate methods
  - Modal truncation [5,12]
  - Component mode synthesis by Craig–Chang [12,13]
  - Krylov subspace methods [14,15]
  - Balanced truncation [16,17]
- Hybrid methods
  - Component mode synthesis by Craig–Bampton [12,18]
  - Component mode synthesis by MacNeal [19]
  - Component mode synthesis by Rubin [20]

The methods just referred to, except for the Krylov subspace methods and balanced truncation, which have their origin in control theory and are considered to be modern reduction methods, were developed specifically for structural mechanics. Modal truncation and component mode synthesis by Craig–Chang, Craig–Bampton, Rubin or MacNeal are all mode-based methods, which means that structural eigenmodes of some sort are employed as Ritz basis vectors. In commercial FE software, generalised coordinates are treated as internal dofs and the coupling of substructures is usually realised at the physical dofs by use of Lagrange multipliers [5]. Consequently, if the global model involved is to be analysed and post-processed in commercial FE software, any methods for