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A hydraulic cylinder model for multibody simulations

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ABSTRACT

In this paper we present a model for a linear hydraulic actuator for multibody simulations. In addition to translational degrees of freedom also the cylinder chamber pressures are taken as state variables. In static analysis the chamber pressures are embedded thus a purely mechanical simulation can be performed. In dynamic analysis we end up to a coupled problem where the chamber pressures are separate variables. The sealing friction is taken into account in both static and dynamic analysis. For numerical solution we use a monolithic algorithm and the coupling matrices between the hydraulic and mechanic variables are presented.

1. Introduction

Hydraulic driven working machines are commonplace in the industry of today. The applications can vary from very robust excavators to very precise manufacturing robots but in outline the hydraulic actuators are quite similar in mechanical sense. Simulation of these applications can derive from need of fatigue assessment or simply to determine the peak stresses due to motion. Engineers are also interested in the positioning accuracy of the systems.

In this paper we will propose a new hydraulic cylinder model for dynamic simulations of hydraulic driven multibody systems. The proposed cylinder element couples the hydraulic and mechanical variables in a monolithic way. We introduce the tangent matrices concerning the mechanical system and the hydraulic system but the analytical coupling matrices as well. In addition the cylinder element is implemented with a novel friction model. However, the dynamic friction model is not suitable for static computations. Therefore we derive a static friction model based on the parameters of the dynamic friction for defining the initial state of the coupled system.

Hydraulic cylinder produces linear movement for the multibody system, for instance a lifting boom of a personnel carriage. The movement is then controlled by the flow rate lead to the cylinder chambers. However the linear movement for the simulation models is traditionally dealt with constraint equations. The classical formulation for constraint equations can be found in [1] but more modern finite element approach is presented in [2]. Solving systems with constraint equations can be done by utilizing for example the Lagrange multiplier method or penalty method. However, as shown in [3] this method can introduce spurious oscillation to the response of the system.

More sophisticated way to model the hydraulic cylinder is a length controlled rod element where the unstressed length of the element is given as a function of time [4]. The compressibility of the hydraulic fluid can is then taken into account as a reduced Young's modulus of the material attached to the element. However, the length change has to be a predefined function estimated from the stationary state of the hydraulic control system in the actual appliance which is simulated. In addition the length controlled rod element has not an embedded friction model. The friction plays an important role, especially when the extension rates are low and there is a possibility for a stick–slip effect.

The cylinder element presented in this paper changes length as a function of the inbound flow rate. The flow rate induces pressure to the cylinder chamber and this force extends the cylinder. Noting that the cylinder force is a function of the chamber pressures and cylinder displacements we find that the cylinder element is a coupling element.

Friction plays an important role in dynamical analyses by introducing damping to the system. The very basic friction model is the Coulomb friction where the friction force is dependent only on the contact force [5]. However, in systems as the hydraulic cylinder, the friction force is known to be a function of the sliding velocity thus a dynamical friction model is required to capture the velocity dependent properties [6]. Furthermore the stick–slip phenomenon where the sliding is not continuous is an important factor to take into account. In stick–slip phenomenon the friction alternates

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between the static friction and sliding friction. The stick–slip phenomenon and the reduction in friction force from static to sliding can be modeled using approach in [7]. Through the stick–slip effect the extension of the cylinder is not continuous and it can induce high stress peaks to the system. The dynamical friction model however is not suitable for initial state computation because of the velocity dependency of the model. Moreover, the dynamical friction model introduces a new variable to the system. Therefore we derive a friction model for the initial state computations based on the parameters of the dynamical friction model.

In this paper the numerical treatment of the coupled system is treated in a monolithic ways, as it was first introduced in [8]. For time integration multi rate integration is exploited where the time stepping scheme for hydraulic cylinder is different from the one of the mechanical system. Information is however changed during each time step. Also in [9] the system was solved monolithically and different integration schemes were compared however.

In this paper we derive a hydraulic cylinder model for multibody simulations and the paper is organized as follows. First we discuss the equations of motion for the coupled system. From linearized equations of motion for the coupled system we present the tangential matrices for the system and in Section 3 we present these matrices. In Section 3 we also present the cylinder element for initial state computations. In Section 4 we discuss the time integration shortly and in the final section we present three different numerical examples. In the first example we deal with the initial state of the cylinder element with constant load. The second example follows from the first example where we present a simple dynamic simulation. In the final example we study a lifting boom and draw comparison between different linear actuator modeling methods.

2. Equations of motion

In this section we present shortly the equations of motion for the coupled hydromechanical simulation model as well as its linearization. The time integration procedure is then discussed in Section 4.

The Hamilton's principle when only conservative external forces are applied to the system states

$$\int_{t_1}^{t_2} \delta\mathcal{L}(\mathbf{q},\dot{\mathbf{q}})dt = \int_{t_1}^{t_2} [\delta\mathcal{T} - \delta\mathcal{V}]dt = 0 \tag{1}$$

where $\mathcal{T}$ is the kinetic energy of the system and $\mathcal{V}$ is the potential energy. The potential energy consists of the potential of the internal forces and external forces. Vector $\mathbf{q}$ is the generalized coordinates describing the system. From the Hamilton's principle we can write the equations of motion for the mechanical system [10] together with the state equation of the hydraulic cylinder

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} = \mathbf{g}(\mathbf{q},\dot{\mathbf{q}},\mathbf{z},t) \\ \dot{\mathbf{z}} = \mathbf{f}_{\mathrm{cyl}}(\mathbf{z},\mathbf{q},\dot{\mathbf{q}}) \end{cases} \tag{2}$$

where $\mathbf{M}$ is the mass matrix and the vector $\mathbf{g}$ is the sum of external, internal and complementary inertial forces. Vector $\mathbf{z}$ collects the state variables of the hydraulic cylinder. Coupling between the mechanical system and the hydraulic cylinder is visible in the right hand side terms of Eq. (2) where we note the crosswise dependency of the systems.

For the solution of the dynamical system we linearize the state equations yielding a linear set of equations with monolithic coupling

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta\ddot{\mathbf{q}} \\ \Delta\ddot{\mathbf{z}} \end{bmatrix} + \begin{bmatrix} \mathbf{C}_{\mathrm{mm}} & \mathbf{0} \\ \mathbf{C}_{\mathrm{cm}} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \Delta\dot{\mathbf{q}} \\ \Delta\dot{\mathbf{z}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{mm}} & \mathbf{K}_{\mathrm{mc}} \\ \mathbf{K}_{\mathrm{cm}} & \mathbf{K}_{\mathrm{cc}} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{q} \\ \Delta\mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{r}^* \\ \mathbf{s}^* \end{bmatrix} \tag{3}$$

where we identify the tangent matrices for the mechanical system as well as for the hydraulic system. The subscript m refers to the mechanical system and c to the cylinder variables. The notation is as follows: The first subscript refers to the state equation to be differentiated and the latter to the variable with respect the differentiation is taken to.

The mass matrix is the derivative of the kinetic energy

$$\mathbf{M} = \frac{\partial^2\mathcal{T}}{\partial\dot{\mathbf{q}}\partial\dot{\mathbf{q}}} \tag{4}$$

The total mass of the system changes due to the changes in cylinder chamber volumes thus the center of the mass also varies. The damping matrix of the mechanical system is

$$\mathbf{C}_{\mathrm{mm}} = -\frac{\partial\mathbf{g}}{\partial\dot{\mathbf{q}}} \tag{5}$$

The tangential stiffness is then defined as

$$\mathbf{K}_{\mathrm{mm}} = -\frac{\partial\mathbf{g}}{\partial\mathbf{q}} + \frac{\partial(\mathbf{M}\ddot{\mathbf{q}})}{\partial\mathbf{q}} \tag{6}$$

where we note that the latter matrix rises from the inertial forces.

For the cylinder element we obtain the hydraulic tangent as

$$\mathbf{K}_{\mathrm{cc}} = -\frac{\partial\mathbf{f}_{\mathrm{cyl}}}{\partial\mathbf{z}} \tag{7}$$

The off-diagonal matrices in the linearized equations of motion in (3) are the coupling matrices. They rise from the crosswise derivation of the state equations thus yielding in non-square matrices. The coupling matrices are then written as

$$\mathbf{C}_{\mathrm{cm}} = -\frac{\partial\mathbf{f}_{\mathrm{cyl}}}{\partial\dot{\mathbf{q}}} \quad \mathbf{K}_{\mathrm{cm}} = -\frac{\partial\mathbf{f}_{\mathrm{cyl}}}{\partial\mathbf{q}} \quad \mathbf{K}_{\mathrm{mc}} = -\frac{\partial\mathbf{g}}{\partial\mathbf{z}} \tag{8}$$

The right hand side terms $\mathbf{r}^* = \mathbf{g}(\mathbf{q},\dot{\mathbf{q}},\mathbf{z},t) - \mathbf{M}\ddot{\mathbf{q}}$ and $\mathbf{s}^* = -\dot{\mathbf{z}} + \mathbf{f}_{\mathrm{cyl}}(\mathbf{z},\mathbf{q},\dot{\mathbf{q}})$ in Eq. (3) are the residuals of the mechanical system and hydraulic cylinder equilibriums respectively. We also note that the coupling damping matrix $\mathbf{C}_{\mathrm{cm}}$ is an actual damping matrix yielding from the friction model.

3. Cylinder element

In this section we present the hydraulic cylinder element and the state Eq. (2) in detail. First we derive the hydraulic cylinder with the dynamic friction model and treat with the coupling between the cylinder and the mechanical system. In the final section we present the cylinder model for initial state computations.

The hydraulic cylinder element is a rod like element with two nodes and three translational degrees of freedom in each node, see Fig. 1. In addition, the cylinder element introduces extra variables: The chamber pressures and a friction induced variable discussed later. Cylinder piston position is $x_c = L_n - L_0$ where L_0 is the initial length of the cylinder and L_n is the current length.

The force that the cylinder produces can be written in terms of the chamber pressures and the friction force as

$$F_c = p_A A_A - p_B A_B - f_{\mathrm{fr}} \tag{9}$$

where p_A and p_B are the pressures of the plus and minus chamber, respectively. The corresponding areas of the cylinder piston are A_A and A_B. Friction force is denoted f_{fr} .

The unit vector in direction of the axis of the cylinder element is given by using the current nodal coordinates as

$$\mathbf{n}_c = \frac{\mathbf{x}_B - \mathbf{x}_A}{\|\mathbf{x}_B - \mathbf{x}_A\|} = \frac{\mathbf{x}_B - \mathbf{x}_A}{L_n} \tag{10}$$

Now the internal force vector can be expressed as