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# Vibration reduction of a structure by design and control of a bolted joint

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ABSTRACT

Control of vibration and noise can be considered as one of the most relevant technological challenges for the designer. In this paper we propose to control the normal force at a bolted joint part of a structure with the aim to reduce the vibration of this structure. In the first part, we study a 4 degrees of freedom model (dof), for which we show the interest of taking as a criterion for the reduction of vibration the maximum displacement in a given point of the structure. Then we propose two control laws for vibration reduction. The first control law takes the critical frequencies of the structure as input parameters and the second does not need any information about the displacement or the eigenfrequencies of the system. In the second part we present a finite element model of a 3D bolted joint taking into account two types of non-linearity (unilateral contact and friction model) in which we show the risk of loss of rigidity in the case of inadequate friction force. We present also a dedicated method to solve this non-linear problem. In the third part, we propose an alternative solution for reducing the vibration by designing a structure with an add-on bolted joint.



## 1. Introduction

Noise and vibration are often limiting factors in performance of many industrial systems. In many applications, reducing structural vibrations is a very important challenge for the conception. Designers mainly use two strategies to control vibration, the first is to avoid exciting the structure at its natural frequencies, the second is to increase damping in the structure when the resonance is inevitable. This can be achieved by using materials having a large structural damping or by introducing equipment which increase the energy dissipation (shunt piezoelectric [1–3], viscoelastic materials [4,5]).

In structural systems, joints and fasteners are used to transfer loads from one structural element to another. Structural joints can be considered as a source of energy dissipation between contacting surfaces undergoing relative motion. Many studies [6–8] indicated that a joint of friction has a great potential to reduce the levels of structure vibration.

The basic models of friction for bolted joints are classified into phenomenological and constitutive [7]. Phenomenological contact models are based on experimental observations and describe the global relation between the tangential force and the relative displacement in the frictional interface. Constitutive models are based on interface physics in the contact area. They describe the friction phenomena in a local manner, e.g, the relation between traction and displacement. Both phenomenological and constitutive models have been studied extensively in the literature [6–8].

The normal force, due to the tightening of the screw, is very important to control the behavior of the bolted joint and therefore the vibration of the structure [9,10]. It can be done by applying a controlled voltage to a piezoelectric stack.

In addition to the aspect of energy dissipation, the variable normal force can be used to change the global stiffness of the structure, and consequently to get out of the critical zone around the eigenfrequencies of the structure.

Nowadays, although the finite elements method FEM presents a satisfactory modeling of a structure, some parts remain difficult to compute precisely due to the non-linearity of unilateral contact and friction model.

This unilateral contact non-linearity is difficult to deal with because of the abrupt changes of behavior during contact-separation and stick-slide transitions.

To solve numerically this non-linear problem, different approaches have been proposed [11–14]. The traditional resolution methods of the contact problems, by penalty, by Lagrange multipliers or by elements of contact, generally involve a very large numerical cost when the number of conditions is large. Moreover, the size of loading increments must be small to ensure convergence. A robust and easy computational tool to analyze quickly and in a reliable way the complex assemblies of structures was proposed

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in [15,16]. It is based on the LATIN method (Large Time INcrement Method) and on a decomposition of the assemblies in substructures and interfaces.

To study the influence of variable tightening force according to time, we study the test-case of a bolted connection between two structures Fig. 1. The contact between two substructures is made via unilateral contact with friction. The piezoelectric material disc is controlled electrically to modify the tightness of the screw.

This paper is organized as follows: in the first part, the test case Fig. 1 is reduced to a 4 dof model with a global relation between the tangential force and the relative displacement in the frictional interface (phenomenological model). First, we show the relevance of a criterion for the reduction of vibration based on the maximum absolute displacement of a particular point of interest. Then two control laws for vibration reduction are presented and commented.

In the second part, we present a 3D model, solved by finite elements, of the same test case (Fig. 1) by taking into account the friction phenomena in a local manner, e.g. the relation between the traction and the displacement (constitutive model). The risk of loss of rigidity in the case of insufficient friction force is pointed out.

In the third part, to overcome the risk of loss of rigidity we propose a design modification of the structure to ensure both sufficient rigidity and reduction of the structure vibrations.

## 2. Discrete model 4 dof

To study the influence of the tightening of the screw on the amplitude response depending on time, we propose a simplified discrete model (Fig. 2), which consists of two mass $M_1$ and $M_{01}$, representing the mass of the upper beam, and $M_2$ and $M_{02}$ representing the mass of the lower beam. The stiffness traction parameters are $K_{11}$ $K_{12}$ for the upper beam and $K_{21}$ for the lower beam. The contact between the two beams is ensured by a stiffness of bolt $K_{0x}$. The tightening of this bolt induces a friction force via a constant friction coefficient $\mu$. The system is excited by an external harmonic force $F_{ext}$.

### 2.1. Problem formulation

The motion equations of this system are given by:

$$M_1\ddot{X}_1 + K_{11}X_1 + K_{12}(X_1 - X_{01}) = 0 \tag{1a}$$

$$M_{01}\ddot{X}_{01} + K_{12}(X_{01} - X_1) + K_{0x}(X_{01} - X_{02}) = r_a \tag{1b}$$

$$M_{02}\ddot{X}_{02} + K_{21}(X_{02} - X_2) + K_{0x}(X_{02} - X_{01}) = -r_a \tag{1c}$$

$$M_2\ddot{X}_2 + K_{21}(X_2 - X_{02}) = F_{ext} \tag{1d}$$

where $r_a$ represents the friction force between $M_{01}$ and $M_{02}$.

The Coulomb friction model can be represented by:

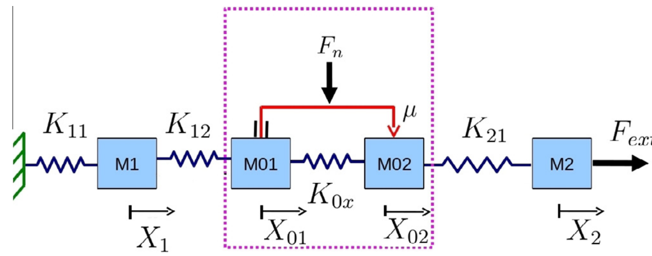


**Fig. 2.** 4ddl Discret model.

$$r_a = \begin{cases} r_y & \text{if } \dot{X}_{02} - \dot{X}_{01} > 0 \\ [-r_y, r_y] & \text{if } \dot{X}_{02} - \dot{X}_{01} = 0 \\ -r_y & \text{if } \dot{X}_{02} - \dot{X}_{01} < 0 \end{cases} \tag{2}$$

with $r_y = \mu F_N$, where $F_N$ is the normal force.

Notice the difference between the two-valuedness of $r_a$ in Eqs. (2a) and (2c) and the multi-valuedness of $r_a$ in Eq. (2b). After substituting $r_a$ from Eq. (2) into the Eqs. (1b) and (1c), it can be seen that the equation of motion has three phases. Therefore to obtain an explicit value of the frictional force $r_a$ when $\dot{X}_{02} - \dot{X}_{02} = 0$, the following development is done: first we define the ratio $\alpha$:

$$\alpha = \frac{M_{01}}{M_{02}} \tag{3}$$

then, subtracting Eq. (1b) multiplied by $\alpha$ from Eq. (1c), we obtain:

$$r_a = \frac{1}{(1+\alpha)}(M_{01}(\ddot{X}_{01} - \ddot{X}_{02})) + F_s \tag{4}$$

with

$$F_s = \frac{1}{(1+\alpha)}(K_{12}(X_{01} - X_1) - \alpha K_{21}(X_{02} - X_2) + (1+\alpha)K_{0x}(X_{01} - X_{02})) \tag{5}$$

Then we define two phases:

- Slipping phase: if $\dot{X}_{01} \neq \dot{X}_{02}$, then $r_a = r_y\, sgn(\dot{X}_{02} - \dot{X}_{01})$
- Sticking phase: if $\dot{X}_{01} = \dot{X}_{02}$ and $\ddot{X}_{01} = \ddot{X}_{02}$, then $r_a = F_s$

Fig. 3 represents the algorithm chart used to solve the system of Eqs. (1a)–(1d).

### 2.2. Approach validation

To validate our approach, we compare our results with analytical ones given for a single-degree-of-freedom oscillator in [17]. The equation of motion of the oscillator is

$$m\ddot{x}(t) + kx(t) + r_a(t) = F_{ext} = p_0 \sin \omega_d t \tag{6}$$

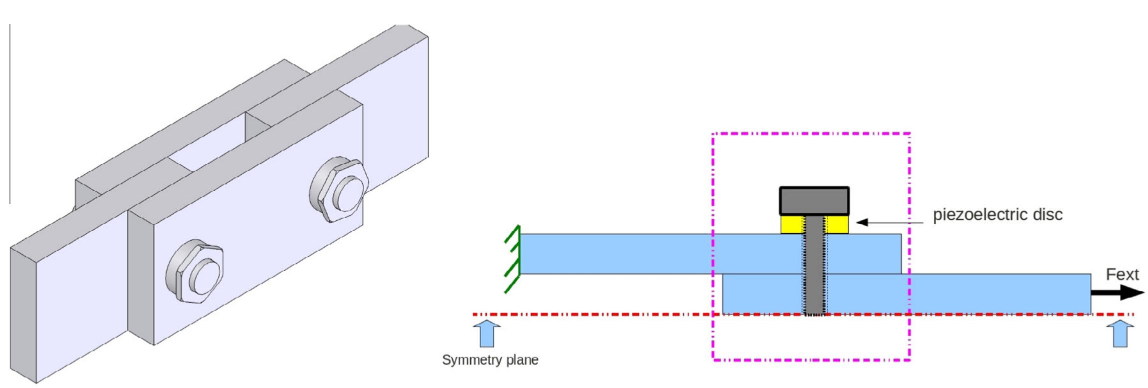


**Fig. 1.** Bolted joint with a piezoelectric disc.