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An error diffusion based method to generate functionally graded cellular structures

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ABSTRACT

The spatial variation of cell size in a functionally graded cellular structure is achieved using error diffusion to convert a continuous tone image into binary form. Effects of two control parameters, greyscale value and resolution on the resulting cell size measures were investigated. Variation in cell edge length was greatest for the Voronoi connection scheme, particularly at certain parameter combinations. Relationships between these parameters and cell size were identified and applied to an example, where the target was to control the minimum and maximum cell size. In both cases there was an 8% underestimation of cell area for target regions.

1. Introduction

Cellular structures, also commonly described as lattice structures, consist of a number of connected members or tessellated unit cells that form a complex structural network. Modern manufacturing techniques, such as additive manufacturing (AM), enable the fabrication of highly complex small to medium sized parts which can include cellular structures. Large scale truss structures are commonly seen in architectural applications where strength and stiffness can be combined with freedom of design. Smaller scale structures have other potential applications such as tailored impact absorption capacity [1–3] and heat dissipation [1,4–11]. An example of such a structure manufactured using an AM process: selective laser melting (SLM) is shown in Fig. 1. This demonstrates a regular tessellation of a unit cell connected to an outer skin. The outer skin can be useful to provide mating faces for adjacent parts, to provide protection for the cellular structure and for maintenance reasons. In this case, a solid outer skin has been used; however, other forms of skin may also be used, where appropriate, such as a 'net' skin [2,3].

Compared to many other manufacturing methods, AM has a comparatively inexpensive cost of complexity [12], and this can actually decrease as part complexity increases as the necessity for support structure for SLM can reduce. The design of these structures, however, remains a challenge. With increased geometric complexity comes increased design complexity, and this is exacerbated when including computational analysis methods and mathematical optimisation techniques in the design process [13–15]. Hence, there is a general requirement for efficient methods for the design and analysis of cellular structures.

This paper focuses on the problem of generating a cellular structure with a spatial variation in the cell size. Different regions of a component generally require different extents of internal structure support. With a fixed, uniformly tessellated cellular structure, this variation can only be achieved by varying the structural member dimensions, which leads to a large number of design variables. As the variation in functionality can only be achieved by varying structural member dimensions, this is restricted by the unit cell size defined prior to optimisation. In addition, some efforts have been made to vary the individual cell design such that the structure is constructed of differing cells [16]; however, this has many challenges and still does not address the cell size variation. This leads us to consider how the design space could be explored more fully by allowing variation of the cell sizes. Some existing cellular structure design approaches do exhibit different sized cells as a by-product of requiring the structure to conform to a shape with curved faces; instead of trimming tessellated cells to this shape, they are either swept [17] or based upon an unstructured mesh [18]; however, these variations are slight.

A novel method is proposed that employs dithering (also known as half-toning) method to achieve spatial variation of cells and subsequent control over component behaviour. Learning from developments linking dithering to meshing techniques [19–22],

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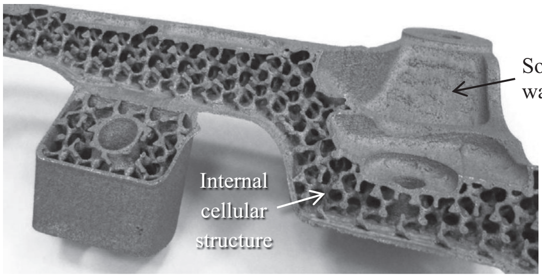


Fig. 1. Cross section through an additively manufactured metal part containing a regular cellular structure.

the dithering defines spatially varying points which are then connected with structural members. These points determine the spatial variation of the cell sizes (area/volume). This paper first introduces the error diffusion methodology used for dithering, and then applies this to some examples to demonstrate the functionality using two point connection schemes. The relationships between the method parameters and the resulting cell size are investigated and their use for controlling cell size and distribution demonstrated.

2. Methodology

2.1. Error diffusion methodology

Dithering converts a continuous tone image into a binary (white–black) representation. This is useful for bi-level printers and displays. When viewed from a certain distance, the binary representation appears similar to the continuous representation to the human eye. There are several dithering methods, which can be divided into two categories: ordered dither and error diffusion [23]. Error diffusion was used for this work as it does not exhibit undesirable grid-like patterns in the dithered result while ordered dithering does [24]. Error diffusion uses an adaptive algorithm based on a fixed threshold to produce a binary representation of the original input.

Consider a 2D array of values where each array location represents a pixel colour value. Each pixel value in the greyscale array, p^i_{xy}, is compared to a predefined threshold value, t, which in this case was 127. The difference determines whether the corresponding pixel in the binary array, b^i_{xy} should be represented by either a white or black value:

$$b_{x,y} = \begin{cases} 255, & \text{if } p_{x,y} > t \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where 255 = white, 0 = black.

The value difference between b_{xy} and p_{xy} is termed the 'error', e_{xy}, and is calculated as:

$$e_{x,y} = p_{x,y} - b_{x,y} \tag{2}$$

The values of adjacent pixels in p are modified based on this thresholding error using a predefined filter (the error is being diffused, hence the name of the method):

$$\begin{bmatrix} p_{x+1,y} \\ p_{x+1,y+1} \\ p_{x,y+1} \\ p_{x-1,y+1} \end{bmatrix}^{i+1} = \begin{bmatrix} p_{x+1,y} \\ p_{x+1,y+1} \\ p_{x,y+1} \\ p_{x-1,y+1} \end{bmatrix}^{i} + e_{x,y} \begin{bmatrix} f_{x+1,y} \\ f_{x+1,y+1} \\ f_{x,y+1} \\ f_{x-1,y+1} \end{bmatrix} \tag{3}$$

where f are the error diffusion fractions from each corresponding location of the 2D filter, specifically 7/16, 1/16, 5/16, 3/16, respectively (see Fig. 2a).

Following this update, the procedure is repeated for each subsequent location in turn. The proportions of the error diffused to adjacent pixels are determined heuristically and a typical filter for this in 2D is shown in Fig. 2a [25]. An empty location in the filter represents zero error diffused to this pixel.

3D error diffusion, while less common than 2D (as there are fewer potential applications), is a simple extension of the 2D process, where the error is diffused in 3D space to voxels around the current voxel. Again, the proportions of the original error to diffuse are determined heuristically and the filter used in this work is shown in Fig. 2b [23]. In this case, Eq. (3) will consist of 20 locations of P_{xyz} corresponding to the locations and fractions shown in Fig. 2b. In the 1D case, the error is diffused completely to the following pixel.

Fig. 3 presents a coarse numerical example that shows the actual value changes resulting from following this procedure for a 2D case. A better visual binary representation would be possible with a higher resolution array. The application of 1D and 2D dithering methods to enable design of a cellular structure with variably sized cells is demonstrated with a larger example problem in the following section.

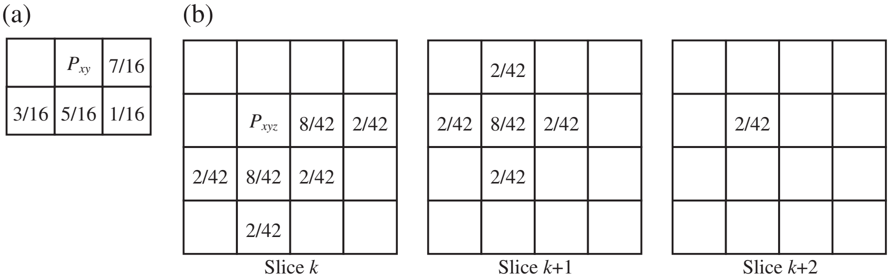
2.2. Cellular structure generation methodology

The method can be split into 3 main stages:

(1) Definition of functional grading.
(2) Error diffusion to generate dithered points of boundary and area.
(3) Application of connection scheme to generate structure cells.

2.2.1. Definition of functional grading

To generate a variable cellular structure, a continuous representation of the desired functional variation is first required. This could be a result of a finite element analysis (FEA), for example, providing a temperature (for thermal problems) or stress (for structural problems) distribution, or a result of a density based



(a)

	P_{xy}	7/16
3/16	5/16	1/16

(b)

Slice k

	P_{xyz}	8/42	2/42
2/42	8/42	2/42	
	2/42		

Slice k+1

	2/42		
2/42	8/42	2/42	
	2/42		

Slice k+2

	2/42		

Fig. 2. (a) 2D filter proposed by Floyd and Steinberg [25] and (b) 3D filter proposed by Lou and Stucki [23] for diffusing the binary thresholding error to adjacent pixels or voxels by the fractions specified, where P_{xy} and P_{xyz} is the current pixel or voxel, respectively.