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A multi-objective optimization approach to the parameter determination of constitutive plasticity models for the simulation of multi-phase load histories

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ABSTRACT

This work develops a multi-objective optimization approach to determine the material constants of constitutive plasticity models. The process is implemented using an actual set of experimental results in the form of multi-phase cyclic-monotonic load histories. With the result of a multi-objective approach being a set of solutions (a set of calibrated models), the characteristics of these solutions are examined individually and within the framework of the entire set. Two elements of parameter determination are considered to address the characteristics of the calibrated models in a multi-objective space: the number of material constants, and the load history used to calibrate the models.

1. Introduction

Constitutive plasticity models have become more and more complex over the past decades. Researchers have strived to develop more advanced formulations in order to simulate more complex behavior of material. One main feature of constitutive plasticity models is the existence of a yield surface defined in stress space wherein its location as well as its size can evolve as a result of plastic deformation. The location and size of the yield surface is designated by means of two distinct classes of internal variables whereby each variable evolves according to a pre-defined rule, namely the kinematic and the isotropic hardening rules. As a result, these two internal variables can be viewed as variables which have a memory of the past loading history. While the kinematic variable and kinematic hardening rule are intrinsic to models focusing on cyclic behavior, inclusion of the isotropic variable and isotropic hardening rule becomes essential for models pertaining to the monotonic or the high amplitude cyclic behavior of a material.

Each and every available hardening rule has been proposed with the intent to allow a more accurate simulation of the materials response. Among all, the fundamental concept of a multi-component hardening rule proposed by Chaboche [1] has proven to lead to significant improvements [2–4]. On the other hand, the comparison of different models and their accuracy in reproducing specific material responses has also been a topic of interest [5–7].

While more advanced hardening rules provide the means for simulating more complicated behavior, they have also caused a fundamental problem, which is the increased number of material constants. These constants must be efficiently selected in order to take advantage of the potential of the model. This has given rise to another area of research regarding constitutive modeling, that is the calibration of a model's parameters.

Calibration techniques can be divided into two main categories. The first category includes those methods that are suitable for hand calculations. The methods in this category are developed for a specific model or at best, a specific class of models. Moreover, they are usually based on assumptions for the type of behavior (monotonic, cyclic, ratcheting, etc.) and initial conditions. Examples of this category of calibration techniques can be found in [5,6,8–11].

On the other hand, the second category involves techniques which are primarily developed for computer calculations. These methods provide a general procedure applicable to any model. While the first category is suitable for, and applicable to, models with few material constants, the second category is applicable to any model and suitable for models with higher number of material constants.

When the number of constants incorporated in a model rises, they effectively lose their clear physical meaning. Hence, they must be viewed from a mathematical perspective [12,13] and selected

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using mathematical methods, such as optimization techniques. Mahnken and Stein [14] successfully employed a gradient-based method to develop a parameter determination method for inelastic models. Saleeb et al. [15] used a gradient-based optimization method in order to minimize the error function and determine optimal material parameters. Also using a gradient-based optimization method, Desai and Chen [16] demonstrated the advantages of determining material constants through optimization, as opposed to the simple method of averaging. Simoni and Schreer [17] implemented a constrained optimization technique to effectively calibrate generalized plasticity models with a large number of material constants. Furukawa et al. [18] developed an automated parameter determination method for constitutive models using an evolutionary optimization algorithm. Yoshida et al. [19] employed sequential quadratic programming to perform a multipoint approximation technique and hence minimize the difference between experimental results and numerical simulations. Rahman et al. [20] used a variation of the genetic algorithm method to determine the material properties of their model. Their work was later extended by Krishna et al. [21] to simulate cyclic and monotonic behavior with one set of material parameters through the imposition of a number of constraints on the model. Chaparro et al. [22] employed different techniques, namely the genetic algorithm as well as a gradient-based method. They demonstrated that a hybrid of these two methods can be more efficient. Rokonuzzaman and Sakai [23] evaluated different genetic algorithm optimization techniques for the determination of material parameters. Yun and Shang [24] effectively used the non-gradient-based Nelder–Mead method in their work. De-Carvalho et al. [25] implemented gradient-based as well as evolutionary algorithms to determine the parameters of their constitutive models. They compared the performance of each method and also proposed improved techniques to increase the efficiency and robustness of the parameter determination process.

While the subject of parameter determination has been given attention in the past decades, some of its issues are yet to be addressed. One issue is the absence of a mathematically sound approach to which parameters can be determined with the intention to simultaneously yield suitable results for more than just one type of loading. It has been shown that if a model is calibrated using the response under a specific type of loading, that model will most likely fail to accurately simulate behavior under other types of loading. This feature has been demonstrated by Bari and Hassan [26], Chen et al. [27], Hassan et al. [28], and Abdel-Karim [11].

In response to the above-mentioned shortcomings, the present work considers the development of an automated method, which based on the concept of multi-objective optimization, provides a mathematically sound solution to the parameter determination problem. The proposed process falls in the category of computerized calibration techniques, indicating its independence to any specific class of material models. Moreover, since it is based on well-established mathematical theories, the method is inherently robust. These are all demonstrated through its application to an actual set of experimental data obtained from a number of rate-independent, isothermal experiments conducted on grade 300 structural mild steel. The important feature of the experimental program lies in the fact that each test involved a multi-phase loading history, whereby strain-controlled cyclic loading was followed immediately by monotonic tensile loading. Consequently, this requires one single numerical model to be capable of correctly simulating the behavior under both phases of loading. This work also aims to investigate the process of using optimization methods to calibrate a constitutive plasticity model to have such capabilities. For this purpose, a yield surface coupled with a multi-component nonlinear isotropic/kinematic hardening rule is considered. As for the optimization method, the gradient-free Nelder–Mead method is employed. This method, otherwise known as the downhill simplex search method, originally developed by Nelder and Mead [29], not only has the advantage of not requiring the derivatives or Hessian of the problem at hand, but also the feasibility of its implementation as a computer code and its high convergence rate.

2. Experimental program

2.1. Material

All the specimens used in this study were made of grade 300 steel taken from the flange of 200UB22.3 hot rolled sections BHP [30]. The preparation of the samples is consistent with the requirements of ASTM E21–92 [31], ASTM E606–92 [32]. The specimen geometry is illustrated in Fig. 1. Both faces of the specimens were grinded to give uniform thickness and a smooth finish across the entire surface. Specimens were restrained by means of two 12 mm high strength bolts at each end. The mechanical properties of the material are given in Table 1, where E is the elastic modulus and σ_y, ε_y, σ_u and ε_u denote the yield stress, yield strain, ultimate stress and ultimate strain (corresponding to the ultimate stress), respectively.

2.2. Cyclic loading

For the cyclic phase, a specifically built fixture was used to prevent the specimens from buckling about the weak axis. This fixture was comprised of two 12 mm plates with aligned holes in order to be fastened by appropriate bolts. The inner faces of these two plates, which would be in contact with the specimen, were polished and greased to yield minimum friction. Loads were applied by an Instron test machine (model 5892) with a capacity of 100 kN. Axial force values were attained by the machine's built in transducer, while strains were accumulated by a non-contact MTS laser extensometer (model LX1500). Tests were carried out in displacement control, however, the displacement rates were adjusted such that strain-rates never exceeded $10^{-4}\,\mathrm{s}^{-1}$, which guarantees rate-independent behavior. Fig. 2 illustrates the three different loading histories applied to the samples. Each of these

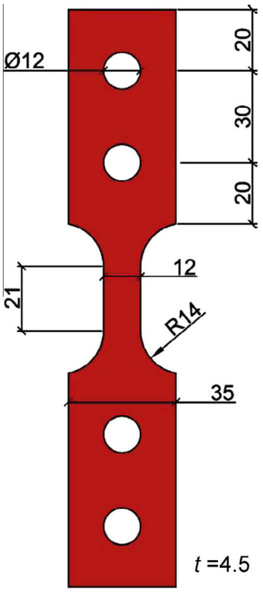

Fig. 1. Sample geometry (in mm).