Computers and Structures 138 (2014) 162–171

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Natural frequencies describe the pre-stress in tensegrity structures

Nasseradeen Ashwear, Anders Eriksson *

KTH Mechanics, Royal Institute of Technology, Osquars backe 18, SE-100 44 Stockholm, Sweden

ARTICLE INFO

Article history:
Received 7 June 2013
Accepted 13 January 2014
Available online 11 February 2014

Keywords:
Tensegrity
Pre-stress
Resonance spectrum
Health monitoring
Buckling

ABSTRACT

This paper investigates the effect of pre-stress level on the natural frequencies of tensegrity structures. This has been established by using Euler–Bernoulli beam elements which include the effect of the axial force on the transversal stiffness. The axial-bending coupling emphasizes the non-linear dependence of the natural frequencies on the pre-stress state. Pre-stress is seen as either synchronous, considering a variable final pre-stress design or as tuning, when increasing pre-stress is followed in a planned construction sequence. It is shown that for a certain tensegrity structure, increasing the level of pre-stress may cause the natural frequencies to rise or fall. This effect is related to whether the structural behavior can be seen as compression or tension dominant. Vanishing of the lowest natural frequency of the system is shown to be related to the critical buckling load of one or several compressed components. Modes of vibration show that when the force in the compressed components approaches any type of critical buckling load, this results in lower vibration frequencies. The methods in this study can be used to plan the tuning of the considered tensegrity structure towards the design level of pre-stress, and as health monitoring tools.

1. Introduction

Over the last four decades, tensegrity structures first invented by Fuller [1] have been extensively studied. They are efficient in some applications, such as certain space applications where the flexibility of shape is important. On the other hand tensegrity structures in some applications may be considered as inefficient [2]. They are used in a wide diversity of fields like aerospace [3], large space structures [4], civil engineering [5] and robotics [6]. Tensegrity structures are classified as free standing pre-stressed pin-jointed cable-strut systems in which contacts are allowed between the struts [7]. They are said to be class one if the bars do not touch each other and class two if two bars are connected at joints [8].

A fundamental aspect of tensegrities is the stress unilateral property of the components: cables and bars must be under tension and compression, respectively. Another important aspect of a tensegrity structure is its pre-stress state, which is an initial internal equilibrium state without external loads, and which to a high degree decides the response properties, e.g., the stiffness to external loading. The design of a tensegrity thereby has to include these aspects in addition to the definition of a specific geometry.

In literature, focus lies primarily on the form finding and self-stress state design, [5,9], but only few studies are devoted to the dynamic response and behavior of the tensegrity. Normally, no analytical solution exists for the natural frequencies of tensegrity structures, and investigations must rely on numerical simulations. Furuya [10] investigated the vibrational properties of a tensegrity mast and shows that the natural frequencies increase as the level of pre-stress increases. Sultan et al. [11] conclude that the model dynamic range generally increases with pre-stress. Moussa et al. [12] demonstrate that the lowest natural frequency of the unloaded single and multi-module continuous strut tensegrity systems is a function of the pre-stress forces in the components of these systems.

Tan and Pellegrino [13] tested the non-linear vibration of a cable-stiffened pantographic deployable structure and concluded that it is possible to vary the stiffness of the structure by increasing the pre-stress level up to the point where the joints start behaving as fully clamped. Ali et al. [14] also discuss the importance of the level of self-stress as a design parameter. Their study indicates that the fundamental frequency of a tensegrity footbridge is not directly influenced by the self-stress level, which must be interpreted as limited variations in frequencies within a reasonable range of pre-stress. Their parametric studies show that the natural frequencies are affected by other design parameters such as the cross sectional area of the components: stiffness and vibration properties are connected.

Many of the cited studies state that the natural frequencies of the tensegrity structures increase when the level of pre-stress is increased without addressing the issue to what extent this is true. It is therefore interesting to examine how stiffness, measured from vibration properties, is affected by the pre-stress level, which in a tensegrity introduces both compression in the bars and tension in the cables. According to Greschik [15], a combined set of bars

* Corresponding author. Tel.: +46 87907950.
E-mail address: anderi@kth.se (A. Eriksson).

http://dx.doi.org/10.1016/j.compstruc.2014.01.020
0045-7949/

and cables may have a dominant strut vibration effect or a dominant cable vibration effect depending on the material and geometry of that set; the structural response is obviously a combination of these aspects.

From the mechanical viewpoint, tensegrities are inherently non-linear structures, with response to loading and excitation strongly dependent on the pre-stressing. With a specific designed pre-stressing introduced, by a shortening or lengthening of one or a few components from their unloaded lengths, the response can, however, be seen as linearized around this ideal state, and this is the common simulation approach. The present work primarily focusses on states during assembly, or after a possible fault in the structure. This means that the conditions deviate from the ideal, and simulations must be performed non-linearly. The non-linear dependence of the resonance spectrum on pre-tension is a key feature of the present work. This with a particular interest in the lower part of the spectrum, where several frequencies are often very closely situated, from a measurement viewpoint.

One of the failure causes of tensegrity structures is the buckling of one or several compressed components. Hanaor [16] states that it is desirable to design double layer tensegrity grids for bar buckling as the governing failure mechanism. Murtha-Smith [17] illustrates that a loss of members due to buckling of one or more bars in a critical truss area is more serious. Another failure mode is the loss of pre-tensioning level, due to relaxation of the control components, over shorter or longer time periods. Both these failure modes are affecting the resonance spectrum of the structure, and are detectable from accurate resonance measurements.

In linearized simulations, tensegrities are commonly seen as trusses, and most of the formulations implemented in literature use truss elements for all members in numerical simulations. This gives only vibration modes where several joints are included, as only axial effects are considered. The above-mentioned non-linearity in the response is, however, primarily related to a coupling between axial and bending behavior of the components, where it is well known that, e.g., a string is tuned to its right resonance by the introduction of an axial force. Similarly, the resonance is affected by a compressive force, and the resonance frequency lowered with increasing force magnitude, until the buckling load, where transversal stiffness disappears and infinite non-periodic displacements are obtained. The consideration of the axial force also in compressed components is another new feature of the present work.

The present work seeks the resonance frequencies of the structure as a function of the pre-stressing. In particular, it is shown that frequencies going down will correspond to an instability of the structure. These instabilities can be of different types, local or global, but will regardless of failure mode be manifested by a tangent stiffness matrix of the structure approaching singularity. The resonance modes, expressed as relative displacements in all degrees of freedom, will be indicated by the corresponding eigenvalues approaching zero with increasing pre-stress, and are more or less localized to one or a few components. A frequency approaching zero will most typically indicate that one or several compressed members are approaching buckling. In the considered context, this buckling of a member during the assembly stage will not lead to a dramatic collapse, but primarily manifest itself in a softening and possibly an incapacity of the structure to reach the planned pre-stressing state.

The present work studies the improvement of current models for resonance frequency simulation of tensegrities by introducing the bending behavior of all components, and by a one-way coupling between the axial force and the stiffness. From this, both local and global vibration modes are obtained. The resonance frequencies are seen as non-linearly dependent on the pre-stressing level in the structure, thereby giving a basis for diagnosis of structural conditions from measured frequencies. The new aspects of tensegrity simulations are shown for simple, plane structures but the basic methods are easily used also for more complex structures, as further discussed below.

With respect to applications, the pre-stressing of the structure is seen in two different contexts below. The first context is a design consideration, when the frequencies are evaluated for a final geometry and topology as a function of the pre-stress level. We denote this as *synchronous pre-stressing* of all unstressed lengths of the components. The calculated resonance frequency spectrum can thereby be seen as a signature for the correctly assembled and pre-stressed structure.

The second context is a production viewpoint. In a *tuning pre-stressing*, it is assumed that a design of geometry and pre-stress is chosen, thence all unstressed lengths are known. By shortening or lengthening the designated control components, the designed configuration and pre-stress is gradually reached. The frequencies are thereby functions of the level to which the structure is yet pre-stressed on its route to the designed state. These functions thus start at a state where most of the structure is assembled from unstressed components, and the designed pre-stress state is successively obtained while length-adjusting the control components to (and perhaps, beyond) the design lengths. The corresponding geometry is in general (slightly) deviating from the desired measures. The spectrum of resonance frequencies can then be seen as a progress indicator of the control process.

The two contexts are thereby seen as tools in the light of the well-known damage identification process for aerospace, civil and mechanical engineering infrastructure known as structural health monitoring ('SHM'). A wide range of highly-effective non-destructive assessment tools are normally available for such monitoring [18]. Different indicators are used in structural health monitoring, but vibration health monitoring methods ('VHM') use the natural frequencies as indicators of the change in one or more structural properties [19]. Vibration of civil engineering and aerospace structures can thereby be used as:

- An indication of design requirement satisfaction;
- a quality control tool in the manufacturing process;
- a damage detection tool during the service life of the structure.

Although many authors have focused on the design and form-finding of the tensegrity structures, vibration health monitoring as design or quality control tools has not been reported in the literature. However, VHM techniques were in [20] applied as a damage detection tool. Panigrahi et al. [20] experimentally applied this method for a single module tensegrity structure, and on a tensegrity grid structure as a damage detection tool without any analytical formulation. They conclude that this technique is convenient and cost effective.

In the health-monitoring context, similar methods as the ones discussed in the present paper can be useful tools, when a measured resonance frequency spectrum can be inversely related to the current pre-tensioning state. In this context, where essentially the resonance spectrum is triggered by external excitation, the most visible modes will, however, be the transversal, more or less localized, cable vibration modes, corresponding to bending of the components. This fact emphasizes the assumption in the current work that bending behavior of the components, and an axial-bending coupling are highly useful. This, in turn, focusses also on the representation of joints between components in the structural model, and this is a key issue in the present work.

2. Formulation and method

Tensegrity structures have some members which are under compression (bars) and some which are under tension (cables).