Computers and Structures 127 (2013) 39–52

Contents lists available at SciVerse ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Selective mass scaling and critical time-step estimate for explicit dynamics analyses with solid-shell elements

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ARTICLE INFO

Article history:
Received 21 March 2012
Accepted 17 October 2012
Available online 4 December 2012

Keywords:
Explicit time integration
Selective mass scaling
Solid-shells

ABSTRACT

Explicit integration is often used in highly nonlinear finite element structural dynamics simulations. However, explicit time integration is stable only if the used time-step is smaller than a critical threshold, which can be shown to depend on the smallest geometrical dimension of the finite elements in the mesh. This aspect is particularly critical when solid-shell elements are used for the analysis of thin walled structures, since the small thickness can lead to unacceptably small time-steps. A selective mass scaling technique, based on a linear transformation of the element degrees of freedom, is proposed in this paper to increase the size of the critical time-step without affecting the dynamic response. An analytical procedure is also developed for the computation of the element highest eigenfrequency and estimate of the critical time-step size. The computational effectiveness and accuracy of the proposed methodology is tested on the basis of numerical examples.

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1. Introduction

Thin walled structures appear in many important engineering applications. For effective and accurate computations, structures of this type are mostly analyzed using shell finite elements, belonging to either one of two main categories: shell elements derived on the basis of the classical or degenerate shell concepts (see [1]), in conjunction with the assumption of plane stress state (see e.g. [2–4]); solid-shell elements, directly derived from three-dimensional continuum elements, not using rotational degrees of freedom and allowing for the implementation of fully three-dimensional constitutive laws (see, e.g. [5–7]).

In recent times, solid-shell elements have been studied more and more intensively, since they are claimed to present several advantages over classical shell elements. Common arguments used in the literature in favor of this statement are: more straightforward enforcement of boundary conditions, possibility to incorporate complex 3D material models, no need for complex update algorithms for finite rotations (event though recent advancements, such as the rotation vector parametrization proposed e.g. in Ref. [8] have greatly reduced the complexity of the problem), easy usage in combination with 3D solid elements, possibility to obtain good accuracy in the through-the-thickness stress distribution in laminated composites, thanks to direct modeling of thickness strains. On the other hand, low-order continuum elements, from which solid-shell elements are usually derived, exhibit several types of locking behavior which, however, can now be controlled adopting assumed strain and/or enhanced strain corrections of the element kinematics. This aspect will not be considered in this paper and the reader is referred to the recent literature on the subject (see, e.g. [9–12] and references therein).

Despite these recent achievements in solid-shell formulations, the fact that the thickness dimension is always significantly smaller than the in-plane dimensions unavoidably leads to a high ratio of transverse to in-plane normal stiffnesses, with a high finite element maximum eigenfrequency and, hence, stiffness matrix ill-conditioning. These inherent difficulties are particularly relevant when iterative solvers are used in implicit formulations or when explicit time integration is employed in dynamic analyses. Explicit integration is usually adopted in wave propagation problems. However, even though implicit integration is the most natural approach in the case of inertia dominated problems, because of convergence difficulties which may occur in highly nonlinear problems (such as, e.g. in contact problems), explicit integration is often preferred also in this case.

The present contribution is focused on explicit central difference approaches, which are only conditionally stable. In the undamped case, a stable time-step must satisfy the condition

$$\Delta t \leqslant \frac{2}{\omega_{max}} \tag{1}$$

where ω_{max} is the maximum eigenfrequency of the assembled mesh. One can also show that

$$\omega_{max} \leqslant \omega^{e}_{max} \tag{2}$$

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http://dx.doi.org/10.1016/j.compstruc.2012.10.021

where ω^e_{max} is the highest eigenfrequency of an individual element in the mesh. A physical interpretation of the stability limit can be obtained from the Courant, Friedrichs and Lewy condition which prescribes that the time-step must be smaller than the time required by a dilatational stress wave to traverse the smallest element of the mesh

$$\Delta t \leqslant \frac{L^e}{c}, \quad c = \sqrt{\frac{\lambda + 2\mu}{\rho}} \tag{3}$$

where L^e is the element smallest geometric dimension (element characteristic length), λ and μ are Lame's constants, ρ is the mass density and c is the wave propagation speed. The ratio L^e/c will be henceforth referred to as "element traversal time".

Two provisions have been recently proposed to circumvent the problem of solid-shell ill conditioning: thickness scaling and mass scaling. In the first case [13–15], the stiffness matrix is modified so as to improve its conditioning. In this case the scaling has either to be closely incorporated into the shell formulation, or it is obtained through multiplication by a preconditioner, which is computationally not effective in explicit analyses. In the second case, the mass matrix is modified, so as to reduce the speed of dilatational stress waves traveling through the shell and hence increase the critical time-step in explicit dynamics. In the present contribution a selective mass scaling technique for explicit dynamics analyses employing solid-shell elements is proposed and assessed on the basis of numerical tests.

The simplest mass scaling technique consists of an artificial uniform increase of the material density, which affects in the same way all structural eigenmodes, and therefore leads to unacceptable inaccuracies in most cases. More accurate approaches can be pursued considering that individual finite elements contribute to the lowest structural eigenmodes mainly with the inertia associated to their rigid body modes. In inertia dominated problems, better results can then be obtained by selectively scaling element masses, in such a way that masses associated to element rigid body modes are not modified. A theoretically motivated scaling, which satisfies this requisite, can be obtained by summing to the mass matrix the stiffness matrix multiplied by a scaling parameter [16,17]. This scaling can be shown to selectively reduce the highest structural eigenfrequencies, with little or zero modifications of the lowest ones. The price to pay is that, after the scaling, the originally lumped mass matrix becomes non-diagonal. The discretization of complex structures generally leads to non-uniform meshes, with fine meshes often localized in small regions, which govern the time-step size for the whole system. In this cases, the scaling can be applied to relatively small patches of elements and the non-diagonal mass matrix can be effectively solved for the nodal accelerations using an iterative scheme [18]. In the general case of thin walled structures, however, the uniform small thickness is the dominant geometric factor and the scaling should be applied to the whole mesh, leading to excessive computational costs when the mass matrix has to be inverted in explicit approaches. Other mass scaling techniques are discussed in Hetherington and Askes [19], and Askes et al. [20], all leading to non-diagonal mass matrices.

To avoid loosing the mass matrix diagonal structure, degrees of freedom governing the element rigid body modes should be isolated and the scaling applied to the remaining degrees of freedom only. This is the concept at the basis of the scaling technique usually adopted in classical shell elements [21]. Since the element rigid body modes are governed by the middle plane degrees of freedom, the idea is to increase the masses associated to rotational degrees of freedom only. In solid-shell elements, only displacement degrees of freedom are used, and the technique cannot be used in a straightforward way. The technique discussed in this paper is then based on a linear transformation of the degrees of freedom, which allows to selectively apply the mass scaling while preserving the mass lumping. It is shown how this can be accomplished in a simple and computationally inexpensive way, so that the time-step size turns out to be governed by the element in-plane dimensions only (element in-plane traversal time), independent of the element thickness.

After the mass scaling is carried out, one needs to accurately compute (or to bound from below) the critical time-step. This can be obtained by estimating the maximum eigenfrequency of each finite element in the mesh. Gershgorin's theorem [22] is widely used to compute an upper bound to such an eigenfrequency. However, when selective mass scaling is applied, the bound turns out to be overly large, leading to too small time-steps. For eight-node hexaedron with constant strains (one integration point) and uniform density, Flanagan and Belytschko [23] provided closed form expressions for bounding the maximum eigenfrequency both in undistorted and distorted element shapes. The critical time-step can also be obtained as the time required by a dilatational stress wave to traverse the smallest element dimension which, for solid-shell elements, is always the thickness. When the element is distorted, the characteristic length can be estimated from below on the basis of geometric considerations [24], but this technique can hardly be used in a consistent way in conjunction with selective mass scaling. Alternatively, ω^e_{max} can be found solving analytically the eigenvalue problem. Besides the solutions provided in Flanagan and Belytschko [23], Ling and Cherukuri [25] provided analytical solutions for plane stress and strain four and eight-node quadrilaterals. Comini and Manzan [26] provided analytical solutions, for conduction type problems, for various two and three-dimensional elements, including triangles and tetrahedra.

In the present work, it is shown how the eigenvalue problem of the undistorted parallelepiped with selectively scaled masses can be reduced to a sequence of second or third order polynomial equations. Explicit expressions for ω^e_{max} are provided and several numerical examples are used to test the accuracy and computational effectiveness of the proposed selective scaling technique. It is also shown how the analytical computation of ω^e_{max} can lead to significant computational gains in some circumstances.

2. Selective mass scaling

The dynamic equilibrium equations of the undamped discretized system are given by

$$\mathbf{M}\mathbf{a} + \mathbf{f}^{int} = \mathbf{f}^{ext} \tag{4}$$

where $\mathbf{a}$ is the vector of nodal accelerations, $\mathbf{M}$ is the mass matrix, $\mathbf{f}^{int}$ and $\mathbf{f}^{ext}$ the vectors of equivalent internal and external nodal forces, respectively. The effect of prescribed displacements is assumed to be incorporated in $\mathbf{f}^{ext}$. The implementation of the central difference integration scheme requires that the accelerations are computed at each time-step as

$$\mathbf{a} = \mathbf{M}^{-1}\mathbf{f} \tag{5}$$

where $\mathbf{f} = \mathbf{f}^{ext} - \mathbf{f}^{int}$ and, for effective computations, $\mathbf{M}$ is assumed to be diagonal.

As anticipated in the previous section, the idea is to modify the element mass matrix preserving its diagonal structure and without affecting lower order structural eigenmodes. This can be achieved by scaling masses of individual elements, in such a way that the energy associated to their rigid body modes remains unaltered. In solid-shell elements the thickness is inherently different from the element in-plane dimensions and can be easily identified from the shell configuration. Making reference to the eight-node solid-shell element with lumped masses shown in Fig. 1, the upper and lower surfaces of the element can also be easily identified. If

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