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Anisotropic constitutive model of plasticity capable of accounting for details of meso-structure of two-phase composite material

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ABSTRACT

In this work, we discuss a novel anisotropic constitutive model of plasticity, which can be used to replace the classical phenomenological models for composite materials, like concrete, with marked difference of behavior in tension and compression. The model is constructed from fine scales by making use of the corresponding meso-scale representation of concrete distinguishing aggregate from cement paste. The elastic response and failure mechanisms at this scale are represented by the corresponding unstructured mesh of truss elements, which is shown to be capable of representing a number of fine features of inelastic response, such as the statistical isotropy of elastic response placed in-between the stringent Hashin–Shtrikman bounds, the pronounced difference of behavior in tension and compression, the sensitivity of bi-axial compression strength to the volume fraction of aggregate and the bi-axial fracture energy corresponding to each particular mode of failure. The results of this kind obtained with meso-model are then packed within a new meso-scale model with these enhanced features, which can be used to successfully replace the standard phenomenological models of concrete in ultimate state computations of complex structures providing increased predictive capabilities.

1. Introduction

The non-linear analysis of structural components working under complex loading program is nowadays an indispensable ingredient of a performance based design. Given this current trend, it has become crucial to have accurate tools that improve the predictive capabilities of macroscopic constitutive equations and furnish a precise description of non-linear response in structural analysis. At the structural scale, phenomenological models based on macroscopic quantities such as macroscopic stresses and strains and macroscopic laws placed in the thermodynamical framework are widely used. Considering cement-based materials such as concrete, there is an extensive literature [1] dealing with its phenomenological constitutive behavior modeling [2,3] according to different loading paths, for static or dynamical cases as well as several multi-physics coupling. However, because of their macroscopic nature, these models present difficulties in describing correctly the physical mechanisms (fracture, damage and transport mechanisms) taking place at finer scales and involving macroscopic observations. Moreover in the case of a complex loading program (e.g. non-proportional loading) they require to properly choose the criterion to use in relation with the studied material and the applied loading. This can lead to substantial difficulties when devising a successful identification procedure [4] which would allow to obtain the correct range of all the model parameters. So, their predictive capability is rather limited for very different loading programs with respect to the one which was used in identification. That is why the main goal of this paper is to provide a novel version of a phenomenological constitutive anisotropic model for concrete based upon information coming from finer scales and the corresponding numerical testing. Namely, we first seek to quantify correspondingly the difference in behavior in tension and in compression, as well as the fracture energy accompanying each of different modes of failure. Second, we carry out large number of numerical tests to master the inelastic mechanism's evolution from finer scales and very different loading programs. The resulting constitutive model we propose on numerical testing can be considered as the most appropriate combination of multi-surface models for concrete combining Drucker–Prager (e.g. [5]) for compression stress and Rankine for tensile stress [6] with a damage model describing the localized failure of structure [7,4].

The key idea here is that such a combination is not done in ad-hoc way, but constructed in accordance with a meso-scale representation of concrete distinguishing aggregates and cement paste [8] and the result of computations corresponding to the chosen loading program. Therefore, the proposed model can account for several fine scale imprints such as compressive strength increase as a function of aggregate volume fraction or the fracture energy typical of failure mode. In this manner, we obtain a model capable

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of describing different failure modes, including the phenomena of localized failure which is of great interest for performance based design. This is also the main novelty of the present approach with relation to our recent works in [8,7], which shows how to exploit the meso-scale computations to provide the meaningful parameters for the macro-scale models for localized failure.

The outline of this paper is as follows. In Section 2, we give a brief description of the meso-scale model of concrete first developed in [8] which is adapted to the present goal. In Section 3, we turn to providing a detailed description of a failure surface obtained with meso-scale model computations.

2. Meso-scale model for failure analysis of two-phase quasi-brittle material

In this section, we give a brief description of the meso-model of a typical two-phase composite material, such as concrete. For the readers interested in more details, a complete description of the model, its numerical implementation and a number of illustrative examples of the model predictive capabilities can be found in [8].

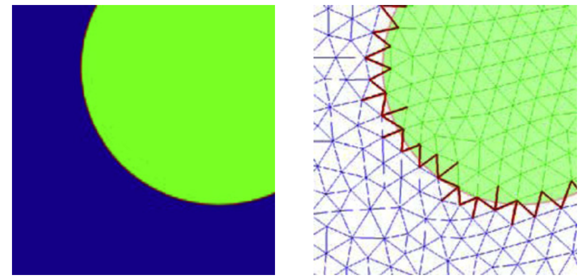
2.1. Meso-model features

The numerical tool in [8] is based upon a two-phase (aggregates melt into a mortar matrix) quasi-brittle finite element model capable of representing the behavior of concrete-like materials under complex loading paths. In order to take into account the influence of the shape, the size, the distribution and the mechanical properties of aggregates on the mechanical behavior of concrete, the meso-scale is chosen to be the scale of computation. This scale has been utilized by others researchers to account for heterogeneities in materials such as concrete [9] and soils [10]. The meso-scale we work with to model two-phases quasi-brittle material is based upon a 3D lattice finite element model [11–14] whose truss elements kinematics is enhanced by two discontinuities.

The first discontinuity is based upon a weak discontinuity (continuous displacement field and discontinuous strain field) [15]. It is introduced because of the retained meshing process which relies on non-conforming mesh [16] where some truss elements are cut into two parts, each having different elastic properties. Considering a two-phase material example in 2D (Fig. 1(a)) with the mortar matrix in blue,[1] one aggregate in green and the interface in red, the finite element discretization gives three sets of truss elements (Fig. 1): those entirely inside the matrix (in blue) with no weak discontinuity activated, those entirely in the aggregate (in green) with no weak discontinuity activated and those split by a physical interface (in bold red) for which the weak discontinuity is activated. Non-conforming meshes provide the advantage to have a meshing process independent from the microstructure (positions of the aggregates and shapes), as well as the fixed size of the mesh in probability studies [15] of meso-structure for this kind of material.

In the present work, this approach is applied to three dimensional domains. Fig. 2 represents a $0.1\ \mathrm{m} \times 0.1\ \mathrm{m} \times 0.1\ \mathrm{m}$ cube with 30% of aggregates. We find again in blue the mortar matrix, in green the aggregates and in red the interfaces.

The second discontinuity relies upon a strong discontinuity (discontinuous displacement field and unbounded strain field) [17–19]. It is introduced in order to represent micro-cracks that may occur in any of different phases (aggregates or mortar matrix for two-phase materials) and to capture the interface failure (debonding). Moreover, the key point pertains to strong discontinuities capability to model softening behavior without any mesh dependency [17,20] which is the major issue dealing with failure of quasi-brittle materials.



(a) Aggregate melt into a mortar matrix (b) FE discretization

Fig. 1. 2D discretization with a non-conforming mesh.

The weak discontinuity is present only for the truss elements split into two parts, each having a different Young modulus. The strong discontinuity is introduced by means of a yield function Φ which is triggered only in traction. Thus two constitutive models appear for a truss element: a continuum one (outside the discontinuity: Fig. 3(a)), which is elastic, and a discrete one (at the discontinuity: Fig. 3(b)) which is quasi-brittle. This approach is called the "Discrete Strong Discontinuity Approach" and can be found in [21]. We denote with t_Γ the traction vector at the discontinuity and with $[|u|]$ the crack opening.

For the discrete model, the softening law is introduced in terms of the internal variable q by considering the exponential form,

$$q = \mathrm{k}([|u|]); \quad \mathrm{k}([|u|]) = \sigma_u\left(1 - \exp\left(-[|u|]\frac{\sigma_u}{G_f}\right)\right) \tag{1}$$

The latter appears in the yield function which can be written as

$$\Phi = t_\Gamma - (\sigma_u - q) \tag{2}$$

In summary, there are altogether eight model parameters:

(1) The Young modulus E_1 for the mortar matrix and E_2 for aggregates for the continuum model.
(2) The ultimate tensile strength before softening, σ_{u_i} and the fracture energy, G_{f_i} ($i = 1, 2, 3$ for respectively the mortar matrix, aggregates and interfaces) for the discrete model. We note that G_{f_i} as the area under the curve $t_\Gamma - [|u|]$.

The mathematical framework for the introduction of these discontinuities in a finite element problem is the Hu–Washizu [22] three fields variational formulation discretized by using the Incompatible Mode method [23]. Among the different possibilities (see [24–26]) to discretized the deformation field, the Enhanced Finite Element Method (E-FEM) [27] has been chosen. Practically, this means that the deformation field is enhanced by two functions: $G_1^{\circ 1/\circ 2}$ and G_2. The first one is a weak discontinuity, introduced as a piecewise linear function over an element capturing the jump of the Young modulus in the deformation field. The second one is split into a constant function and a Dirac function capturing the unbounded nature of the strain field in presence of a strong discontinuity. One can observe these functions on Figs. 4 and 5 thanks to a zoom on a interface truss element (red bold elements on Figs. 1 and 2).

This kind of discretization leads to the following system of equations to be solved:

[1] For interpretation of color in Figs. 1, 4 and 5, the reader is referred to the web version of this article.