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Boundary element analysis of elastic fields in non-horizontally layered halfspace whose horizontal boundary subject to tractions

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ABSTRACT

This paper presents a boundary element analysis of linear elastic fields in a layered halfspace whose material interface planes are not parallel to its horizontal boundary surface. This boundary element analysis uses the generalized Kelvin solution in a multilayered elastic solid (or the so-called Yue's solution) for taking into account the non-horizontally layered structures. It also adopts the infinite boundary elements for evaluating the influence of a far-field region. It further adopts both the discontinuous finite and infinite boundary elements to discretize the boundary surface around the strike lines of the inclined material interfaces. It uses Kutt's numerical quadrature to evaluate the strongly singular integral in the discretized boundary integral equation. Numerical examples are presented to illustrate the effects of the non-horizontally layered materials to the displacements and stresses induced by the tractions on the horizontal boundary surface. Two non-horizontally layered halfspace models are used for numerical analysis and illustrations. Numerical results show that across the material interface, the elastic displacements are non-smoothly continuous to different degrees and some stress components can have very high values at and around the interface planes, which can be important to tensile or shear failure in non-homogeneous materials.

1. Introduction

1.1. Horizontally layered halfspace model

Layered solid materials widely exist in nature or man-made structures. Their responses to external loadings before failure can be modeled with the responses of elastic halfspace model subject to the same external loading. The elastic halfspace model can compose of the layered elastic materials with the same mechanical properties and geometries of the actual layered solid materials. The interfaces of the layered solid materials may or may not be parallel to their boundary surfaces. Many investigations have been done for the elastic responses of the horizontally layered halfspaces subject to tractions or other types of loading conditions by many researchers since 1940s (e.g., [1–7]). A relatively complete list of relevant publications can be found in the recent publications by Yue [8,9].

1.2. Non-horizontally layered halfspace model

As shown in Fig. 1, this paper examines the elastic responses of an elastic halfspace with non-horizontally layered solid materials due to the action of tractions at the horizontal boundary surface. There is a very limited investigation on the elastic responses of such non-horizontally layered halfspaces subject to tractions. Two examples of relevant studies are given by Almeida Pereira and Parreira [10] and Moser et al. [11], respectively. Due to its importance in science and engineering and its difficulties in analytical or numerical formulations, this paper aims to develop a novel boundary element method to calculate the elastic responses of the displacement and stress fields with high efficiency and accuracy.

1.3. Classical boundary element method and issues

The classical boundary element method (BEM) is ideally suited for the analysis of elastic solid materials occupying either full-space or halfspace because BEM only needs to discretize the external boundary surface and automatically satisfies the conditions at the infinity. The classical Kelvin's solution is commonly used as the fundamental singular solution in the classical BEM. When it is applied to a layered solid as shown in Fig. 1, it needs to divide the layered solid into many homogeneous domains. Furthermore, it needs to discretize the internal interfaces of the layered materials so that the continuity conditions at the internal interfaces can be formulated numerically together with those at the external boundary surfaces. For example, Almeida Pereira and Parreira [10] and

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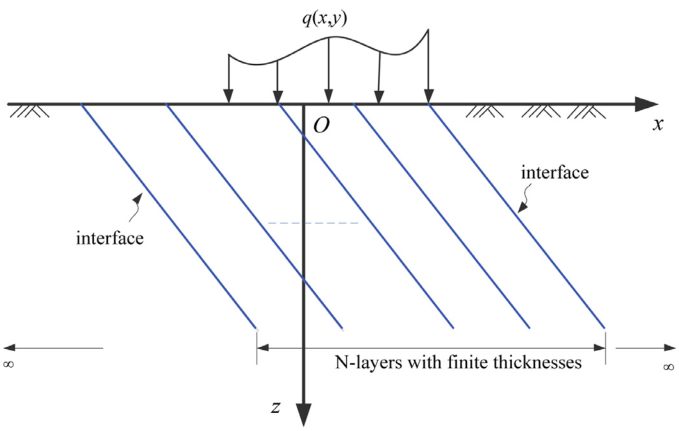

Fig. 1. A layered halfspace subject to distributed loadings on the boundary surface.

Moser et al. [11] adopted this multi-region method of BEMs and analyzed the elastic responses of a halfspace with two non-horizontal layers subject to tractions. As the number of layers increases, however, the efficiency and accuracy of the classical BEM decrease significantly. This multi-region method also has the disadvantage in effective and accurate calculations of the elastic fields across internal material interfaces. Thirdly, the fundamental singular solutions, used as the kernel functions, in the classical BEMs vary rapidly if an internal point is very close to the elements located at material interfaces.

1.4. Aim and approach of this study

In this paper, a single-region BEM is developed and presented for the analysis of the elastic responses of a non-horizontally layered halfspace subject to tractions (Fig. 1). The generalized Kelvin solution in a multi-layered elastic solid given by Yue [12] is used to eliminate the discretization task at the internal interfaces of layered materials. Furthermore, the infinite boundary element technique proposed by Moser et al. [11] is used to take into account the influence of a far-field region because of its straightforward implementation. Other infinite boundary element techniques can be found in the publications by Waston [13], Beer et al. [14], Beer and Waston [15], Zhang et al. [16], Liu and Farris [17], Almeida Pereira and Parreira [10], Davies and Bu [18], Bu [19], Gao and Davies [20], Moser et al. [11], Salvadori [21], Liang and Liew [22] and Ribeiro and Paiva [23]. Thirdly, the discontinuous boundary element technique is adopted to deal with the step-discontinuity of material properties at the interface strike line on the horizontal boundary surface of the non-horizontally layered halfspace. Fourthly, special attentions are given to various singular integrals involved in the discretized boundary integral equations. The proposed BEM is applied to specifically solving the elastic response of a halfspace with two or three non-horizontal layers under a square footing loading on the horizontal boundary surface. Numerical results show the influence of non-horizontally layered materials to the elastic displacement and stress fields induced by the normal footing tractions on the horizontal boundary surface.

2. The governing equations for BEM in non-horizontally layered halfspace

Fig. 2 shows the horizontally oriented boundary surface of the non-horizontally layered halfspace model. The interface strike line represents an intersection line of an internal interface plane of any two fully contacted dissimilar material layers with the horizontal boundary surface. The boundary surface is divided into two parts S_F and S_I. They represent a core region around the traction area and a far-field region beyond the traction area, respectively. Accordingly, using the generalized Kelvin solution of a multilayered solid occupying the full-space

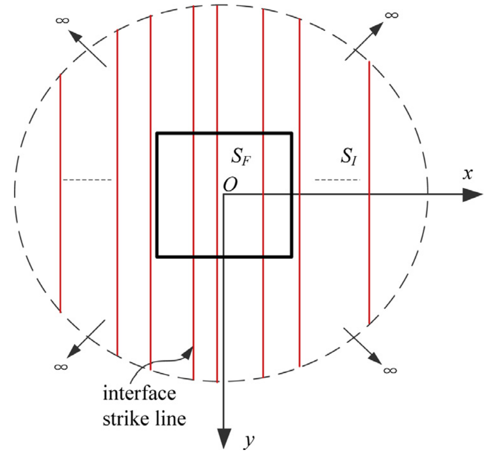

Fig. 2. Definition of S_F and S_I on the boundary surface of a non-horizontally layered halfspace.

[12], the boundary integral equations for the non-horizontally layered halfspace without body forces can be expressed as

$$c_{ij}(P)u_j(P) + \int_{S_F+S_I} t_{ij}^Y(P,Q)u_j(Q)dS(Q) = \int_{S_F+S_I} u_{ij}^Y(P,Q)t_j(Q)dS(Q) \tag{1}$$

where P and Q are, respectively, the field and source points; t_j and u_j are, respectively, tractions and displacements; t_{ij}^Y and u_{ij}^Y are, respectively, the kernel functions of the tractions and displacements of the generalized Kelvin solution.

The free term $c_{ij}(P)$ in Eq. (1) depends only upon the asymmetric behavior of the singular terms of the generalized Kelvin solution and the local geometry of the boundary at the point P. In using the fundamental solution of a layered space, $c_{ij}(P) = 0.5\delta_{ij}$ for the point P located on a smooth boundary and not at the material interface [24]. When the point P is located at the strike line of the material interface on the horizontal boundary surface, there is no closed-form expression for $c_{ij}(P)$ available in the open literature. So, instead, the discontinuous boundary element technique is adopted for resolving this task. Its details are given in Section 4.1.

After obtaining the displacements and tractions on the boundary, the displacements at any internal point p can be determined by using the displacement integral equations as follows

$$u_i(p) + \int_{S_F+S_I} t_{ij}^Y(p,Q)u_j(Q)dS(Q) = \int_{S_F+S_I} u_{ij}^Y(p,Q)t_j(Q)dS(Q) \tag{2}$$

By using Eq. (2), the strain–displacement equations and the constitutive equations, the stresses at any internal point p can be expressed as

$$\sigma_{ij}(p) = \int_{S_F+S_I} U_{ijk}^Y(p,Q)t_k(Q)dS(Q) - \int_{S_F+S_I} T_{ijk}^Y(p,Q)u_k(Q)dS(Q) \tag{3}$$

where U_{ijk}^Y and T_{ijk}^Y are the new kernel functions obtained from the displacements and stresses of the generalized Kelvin solution in a layered solid of full-space extent and the relative functions are presented in Appendix A.

3. The generalized Kelvin solution in non-horizontally layered full-space

3.1. Two Cartesian coordinate systems and relationship

As shown in Figs. 1 and 2, the boundary integral equations are established in the Cartesian coordinate system *Oxyz*, where the non-horizontally layered material interface planes have a dip angle θ to the horizontal *Oxy* plane. On the other hand, another Cartesian coordinate