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Identification of transient boundary conditions with improved cuckoo search algorithm and polynomial approximation

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A B S T R A C T

The cuckoo search (CS) algorithm combined with Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm (CS-BFGS) is proposed to identify time-dependent boundary conditions for 2-D transient heat conduction problems in functionally gradient materials. Firstly the dual reciprocity boundary element method (DRBEM) is used to solve the direct problem. Then taking the unknown boundary conditions as a polynomial function of coordinates with time-dependent coefficients, the CS-BFGS is applied to obtain the unknown coefficients of the polynomial. As a result, the transient boundary conditions are evaluated. The convergence speed of the CS-BFGS algorithm is faster than the CS algorithm. What's more, the effect of the polynomial degree is discussed. As the polynomial degree increases, the inverse results are more accurate but the iterative number and computation time also increase. Finally, the influences of the position and number of measurement points, and random errors on the inverse results are investigated. With the measurement points closer to the boundary, with the increase of measurement point number and with the decrease of measurement errors, the results are more accurate.

1. Introduction

The direct heat conduction problem is concerned with the determination of temperature distribution in a domain when boundary conditions, initial conditions, thermo-properties, heat sources and the geometry of the domain are specified. In contrast, the inverse heat conduction problem involves the determination of boundary conditions, thermo-properties, heat sources, or geometry from the measured temperature information in the domain [1–4].

The boundary element method (BEM) is an important alternative technique for solving the heat conduction problem [5–7]. Compared with other numerical methods, the BEM has advantages of small amount of calculation and high precision. For non-linear, non-homogeneous and transient problems, the domain integral occurs in the resulting integral equations, which makes the BEM lose its advantage of boundary only discretization. The dual reciprocity method (DRM) can transform the domain integral into the boundary integral [8]. The method combining the DRM with the BEM is called the dual reciprocity boundary element method (DRBEM). Up to now, the DRBEM has been used in many fields [9–11].

For the inverse boundary condition problems, many numerical methods have been proposed. Beck [12,13] estimated heat flux by the finite difference method. Busby and Trujillo [14] determined the unknown heat flux by the method of dynamic programming. Falk and Monk [15] studied the Cauchy problem for elliptic equations by the minimal energy technique. Ingham et al. [16,17] investigated the linear and nonlinear inverse heat transfer problems by the minimal energy technique. Lesnic et al. [18] and Mera et al. [19] solved the Cauchy problem for the steady-state heat conduction equation by the BEM. The DRBEM combined with the sequential function specification method was used to identify the unknown boundary heat flux by Behbahani-Nia and Kowsary [20]. Marin used the method of fundamental solutions to deal with the inverse boundary condition problem for steady-state heat transfer by the Tikhonov regularization [21] and singular value decomposition (SVD) [22]. Onyango et al. [23] employed the BEM to solve the missing terms involving the boundary temperature, heat flux and law for the boundary conditions. Chen [24] applied the window function to regularize the divergent problem in the Laplace equation with overspecified boundary conditions in an infinite strip region. Hon and Wei [25,26] developed a new meshless and integration-free numerical scheme combined with Tikhonov regularization method for solving an inverse heat conduction problem. Li et al. [27] approximated the unknown boundary conditions by the polynomial functions and solved the 2-D Cauchy problem by using the meshless scheme. Fan et al. [28] dealt with the Cauchy problem by the generalized finite difference method. Yeih et al. [29] used the modified Tikhonov's regularization to solve the Cauchy problem of the nonlinear steady-state heat conduction problem. Wang et al. [30] solved the 3-D inverse heat conduction problems by

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Nomenclature

$\mathbf{b}$	polynomial coefficient
c	specific heat
C_i	singularity coefficient
d	degree of the polynomial
$\mathbf{E}$	estimated Hessian matrix
$f(\mathbf{x}, \mathbf{z}^j)$	radial basis function
$\mathbf{G}, \mathbf{H}$	coefficient matrices of BEM
$\widehat{\mathbf{H}}$	Hessian matrix
$J(\mathbf{b})$	objective function
k	iteration number
L	number of boundary nodes
N	number of collocation points
$\mathbf{n}$	unit outward normal to the boundary
$\mathbf{p}(\mathbf{x})$	polynomial
p_a	possibility of finding
q_i^{est}	estimated heat flux
q_i^{exa}	exact heat flux
$\bar{q}$	prescribed normal derivative of temperature
R	distance between observation and collocation points
S	number of inverse nodes
t	time variable
$\mathbf{T}$	temperature vector
$\mathbf{T}^c$	exact temperature vector
T_0	initial temperature
$\widehat{T}_i$	computed temperature
$\bar{T}$	prescribed temperature
T_i^{est}	estimated temperature
T_i^{exa}	exact temperature
$T(\mathbf{x}, t)$	temperature
$\hat{T}(\mathbf{x}, \mathbf{z}^j)$	particular solution
u	normal stochastic variable
$\mathbf{w}$	random variable vector
x_i	i-th component of coordinates
$\mathbf{x}$	coordinate
$\mathbf{z}^j$	j-th collocation point

Greek symbols

α^j, γ^j	unknown coefficients
Γ	boundary of the domain
$\Gamma(z)$	gamma function
Δt	time step
ε	convergence criterion
θ	polar coordinate angle
$\lambda(\mathbf{x})$	thermal conductivity
μ	step size
ξ	a number with the range [0,1]
ρ	density
σ	standard deviation
φ	Lévy exponent
χ	regularization parameter
Ω	a bounded region
∂_0	step size parameter

the truncated SVD. Su et al. [31], Singh et al. [32] and Duda [33] identified the time-dependent heat flux by the Levenberg-Marquardt algorithm. Cui et al. used the Levenberg-Marquardt algorithm to determine the damping factors [34] and the multi-parameters of boundary heat flux [35]. The conjugate gradient method (CGM) is the most effective method for solving the boundary condition identification problems. Alifanov and Mikhailov [36,37] determined the unknown heat flux by the CGM. Hào and Lesnic [38] solved the Cauchy problem for the Laplace equation by the CGM. Huang et al. [39,40] estimated the surface heat flux in 3-D heat conduction problem by the CGM. Singh and Tanaka [41] applied the DRBEM and preconditioned CGM to estimate heat flux. Bozzoli and Rainieri [42] reconstituted the heat flux density distribution in 2-D steady-state heat conduction problem by the CGM. Mohammadiun et al. [43] applied the CGM to estimate the time-dependent heat flux. Yang and Chen [44] identified the unknown time-dependent heat flux of the disc by the CGM. Mohebbi and Sellier [45] used the CGM to identify the thermal conductivity, heat transfer coefficient and heat flux in 3-D problem based on the finite difference method. It should be concluded that the CGM can solve the inverse problems efficiently, but it is a locally convergent method and the results are sensitive to the initial values.

Recently, the stochastic optimization methods have become popular means for solving the inverse problem due to their capability of finding the global optimal without computing the complicated gradient of the objective function. Up to now, most of the numerical works for resolving inverse heat transfer problems are based on genetic algorithms (GA), particle swarm optimization (PSO) and other metaheuristic algorithm. Karr et al. [46] solved the inverse initial value, boundary value problems by the GA. Vakili and Gadala [47] identified the heat flux by using the PSO algorithm. Liu [48,49] estimated the surface heat flux in 3-D heat conduction problems based on the PSO. Wang et al. [50] applied the double decentralized fuzzy inference method to estimate the time and space-dependent thermal boundary conditions.

Based on the interesting and rather awkward way of survival of the cuckoo species, Yang and Deb [51] proposed the cuckoo search (CS) algorithm. The CS algorithm has advantages of good global search capability and few control parameters. Up to now, the CS algorithm has been widely used in hydrothermal scheduling problems [52], aerodynamic shape optimizations [53] and reliability optimization problems [54]. Udayraj et al. [55] compared the PSO, ant colony optimization and CS algorithms for inverse heat transfer problems.

Recently, Zhou et al. [56] used the firefly algorithm and the Newton method to identify the time-dependent boundary conditions for the transient heat conduction problem in homogeneous medium. The unknown temperature without polynomial approximation is directly identified, and the computation dimensionality and time of the inverse problem are very large. In this paper, a polynomial function [27] with unknown coefficients is developed to approximate the unknown temperature and the inverse problem is transformed into finding the unknown coefficients of the polynomial function. In this way, the dimensionality of the inverse problem is reduced significantly. What's more, the CS-BFGS algorithm is proposed to identify boundary conditions for the transient heat conduction problem in functional gradient material.

The structure of this paper is organized as follows: firstly, the direct problem is solved by the DRBEM in Section 2. The unknown boundary temperature is approximated by the polynomial and the objective function is defined in Section 3. The theory and the flowchart of the CS-BFGS are shown in Section 4. Numerical examples are presented in Section 5. Finally, some conclusions are drawn in Section 6.

2. Direct problem

2.1. Governing equation

The governing equation of transient heat conduction problem in functional gradient material can be expressed as

$$\frac{\partial}{\partial x_1}\left(\lambda(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial x_1}\right) + \frac{\partial}{\partial x_2}\left(\lambda(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial x_2}\right) = \rho c\frac{\partial T(\mathbf{x},t)}{\partial t}, t \geq 0, \mathbf{x} \in \Omega \quad (1)$$

and the initial condition is given as