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Update-grid reanalysis method based on NS-FEM for 3D heat transfer problems

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A R T I C L E I N F O

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A B S T R A C T

Repeated modifications and iterative recalculations cost a great amount of computation time in the structural design process. Reanalysis is an efficient computational method that can ensure the accuracy of a solution in place of repeated full finite element analysis (FEA) and other numerical methods. In this study, an update-grid (UG) reanalysis (UGR) method is suggested to analyse three-dimensional (3D) heat transfer problems. Compared with other reanalysis methods, the suggested method easily establishes a mapping between initial meshes and modified meshes using the mapping relationship. Generally, a modified structure can be reanalysed by directly reusing transfer operators from the UR analysis. Therefore, the modified meshes can be solved by constructing a map from the initial meshes, even if the model is totally re-meshed. Moreover, considering the accuracy of the analysis, a node-based smoothed finite element method (NS-FEM) is used as the main solver. To evaluate the performance of the suggested method, several heat transfer problems are investigated. The results demonstrate that the UGR method is more accurate and efficient compared with the popular multi-grid (MG) preconditioned conjugate gradient (PCG) method.

1. Introduction

Heat transfer analysis of modified structures is a key issue in engineering optimization problems. However, the optimization procedure is often accompanied by repeated iterative modifications and recalculations. Therefore, computational cost is expensive in practice. Therefore, an auxiliary computational method, reanalysis, has been proposed to substitute full finite element analysis (FEA) during optimization. In the past decades, several reanalysis methods have been suggested and applied in multiple disciplines. Generally, reanalysis methods can be classified into two categories: direct methods (DMs) and approximate methods. DMs are usually applied to local or low-rank modification and were first proposed by Sherman and Morrison in the 1950s [1]. Subsequently, Woodbury developed a DM for multiple rank-one changes [2]. The SMW formula is an exact reanalysis method suitable for only low-rank modification. For multiple modifications, the SMW formula would be performed repeatedly, and its efficiency would drop. To make reanalysis more feasible, several alternative reanalysis methods were suggested. In the early 2000s, Noor presented the advances and applications of reduction methods for large systems [3]. Chen et al. developed a static displacement reanalysis method based on usual perturbation and Padé approximation [4]. Chen and Wu proposed an eigenvalue reanalysis method based on the Neumann series expansion and epsilon-algorithm [5]. Yang et al. presented a multi-sample compression algorithm for elastoplastic FEM [6]. Song et al. proposed a novel reanalysis algorithm differing from the Woodbury formula based on updating the triangular factors of the original stiffness matrix [7]. For large-scale problems, Huang and Wang proposed the independent coefficient (IC) method, which only requires initial responses without a decomposition operator [8]; they also developed an indirect factorization updating (IFU) method for boundary modifications [9]. Gao and Wang presented a reanalysis method based on block matrices to handle large modification problems [10]. For high-rank problems, approximate methods are widely used. Of those methods, combined approximation (CA) may be the most popular approximate reanalysis method [11,12].

In the early 1990s, the first study on the CA method for structural optimization problems was proposed by Kirsch [13]. Sequentially, Kirsch presented a unified approach for all types of topological modifications [14]. CA has been widely used in multiple disciplines, such as structural static reanalysis [11,15], nonlinear dynamic reanalysis [16,17], and topological optimization [18], due to its feasibility. Based on CA, other alternative methods were developed. For example, Wu et al. integrated reanalysis and PCG to both add and remove degrees of freedom (DOFs) [19,20]. Based on residual increment approximations, Materna et al. developed a nonlinear reanalysis method [21]. For heat transfer

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Abbreviations

FEA	finite element analysis
UG	update-grid
UGR	update-grid reanalysis
GMG	geometric multi-grid
DOF	degrees of freedom
BEM	boundary element method
NS-PIM	node-based smoothed point interpolation method
NS-FEM	node-based smoothed finite element method
AMG	algebraic multi-grid
DM	direct method
IC	independent coefficient
MG	multi-grid
CA	combined approximation
IFU	indirect factorization updating
MGPCG	multi-grid preconditioned conjugate gradient method
PCG	preconditioned conjugate gradient

problems, Feng et al. used the CA method to solve transient nonlinear dynamic heat conduction problems [22]. Wang et al. extended CA and IC for heat condition problems [23]. To solve the large-scale CAE model, some efficient parallel reanalysis methods were developed based on GPU platforms [24–26].

Theoretically, reanalysis can be employed for different kinds of solvers, such as FEM [3,26], the meshless method [27,29,30] and the boundary element method (BEM). However, FEM has inherent weaknesses known as the overly stiff phenomenon and the lower bound solution. Therefore, some gradient smoothing techniques were developed to solve these problems [28–31]. According to the "softened" behaviour, SFEM has been proved to produce upper bound solutions for heat transfer problems [28–35]. For example, a node-based smoothed point interpolation method (NS-PIM) was developed to analyse steady-state thermoelastic problems [36]. NS-PIM was applied to solve three-dimensional (3D) heat transfer problems with complex geometries and complicated boundary conditions [28]. In addition, NS-FEM was extended to solve acoustic problems by Wang [37]. Therefore, NS-FEM is used as the main solver of reanalysis in this study.

For reanalysis, mesh consistency is the main limitation for practical applications. Most existing reanalysis methods require high consistency between initial meshes and modified meshes. This means the nodal locations of initial meshes and modified meshes should be determined before reanalysis. Therefore, to avoid these operations on meshes, the UGR method was developed in this study. The update-grid method mainly uses two grids (the initial and modified meshes) directly without any preprocessing, similar to the multi-grid (MG) method. Moreover, compared with other methods, the most significant advantage of MG is asymptotic convergence [38]. MG methods are mainly divided into two categories: geometric multi-grid (GMG) and algebraic multi-grid (AMG). Compared with GMG, AMG only uses virtual meshes generated by algebraic methods (rather than an extra geometric mesh). In previous studies, Amir used MG in topology optimization by combining it with approximate reanalysis [39] and offered significant improvements with respect to the original contribution. Huang et al. also integrated MG and reanalysis for fast structure analysis [40].

The rest of this paper is organized as follows. The basic theories of heat transfer are briefly described in Section 2. The basic theories of NS-FEM and UGR are presented in Section 3. In Section 4, four cases demonstrate the performance of the UGR method. Finally, summaries are given.

2. Thermal governing equations and boundary conditions

The equilibrium equation of three-dimensional heat transfers problem can be written as:

$$\left(k_x\frac{\partial^2 T}{\partial x^2}+k_x\frac{\partial^2 T}{\partial y^2}+k_x\frac{\partial^2 T}{\partial z^2}\right)+Q(x,y,z)=\rho c\frac{\partial T(x,y,z)}{\partial t} \tag{1}$$

where $T(x, y, z)$ is the temperature at time t, $Q(x, y, z)$ is the rate of internal heat, ρ is density, c is specific heat and k_x, k_y and k_z are the heat conductivities in the x, y and z directions, respectively. The boundary conditions that are considered are as follows:

$$\text{Initial boundary}:\quad T(x,y,z,t=0)=T_0 \tag{2}$$

$$\text{Dirichlet boundary}:\quad T|_{\Gamma}=T_k \tag{3}$$

$$\text{Neumann boundary}:\quad -k\frac{\partial T}{\partial n_o}\bigg|_{\Gamma}=q_o \tag{4}$$

$$\text{Robin boundary}:\quad -k\frac{\partial T}{\partial n_o}\bigg|_{\Gamma}=h_c\left(T-T_\infty\right) \tag{5}$$

where T_0 is the initial temperature, T_k is the known temperature, T_∞ is the environmental temperature, q_o is the prescribed heat flux, h_c is the convection coefficient, n_o is the outward normal direction of the boundary and Γ represents the boundary.

3. NS-FEM reanalysis

Reanalysis is an auxiliary solver to improve the efficiency of a main solver. Generally, most reanalysis methods are based on FEM. However, traditional FEM has been deemed "overly stiff" numerically. Therefore, SFEM is used in this study. Among various SFEM models, NS-FEM has some important properties, such as the following:

(1) It is softest and possesses an upper bound;
(2) It uses linear, triangular and tetrahedral elements;
(3) It can guarantee accurate and convergent properties of solutions; and
(4) The temperature at the node can be calculated directly.

Therefore, NS-FEM is used as the main solver.

3.1. NS-FEM based on tetrahedron elements

As shown in Fig. 1, for a tetrahedron element, the smoothing domain $\Omega^{(k)}$ associated with node k is generated by sequentially connecting the mid-edge points to the centroids of the surface triangles and tetrahedrons.

In NS-FEM, a smooth strain in domain $\Omega^{(k)}$ associated with node k can be created according to the strain $\varepsilon=\nabla_s\mathbf{u}$:

$$\dot{\boldsymbol{\varepsilon}}_k=\int_{\Omega_k}\boldsymbol{\varepsilon}(\mathbf{x})\Phi_k(\mathbf{x})\mathrm{d}\Omega=\int_{\Omega_k}\nabla_s\mathbf{u}(\mathbf{x})\Phi_k(\mathbf{x})\mathrm{d}\Omega \tag{6}$$

where $\Phi_k(\mathbf{x})$ is a given smoothing function that satisfies the least unified properties, $\mathbf{u}$ is the trial function and $\nabla_s\mathbf{u}$ is the symmetric gradient of the temperature field.

$$\int_{\Omega_k}\Phi_k(\mathbf{x})\mathrm{d}\Omega=1 \tag{7}$$

The local constant smoothing function can be given as:

$$\Phi_k(\mathbf{x})=\begin{cases}1/V^{(k)}, & \mathbf{x}\in\Omega^{(k)}\\ 0, & \mathbf{x}\notin\Omega^{(k)}\end{cases} \tag{8}$$

where $V^{(k)}$ is the volume of the smoothing domain $\Omega^{(k)}$:

$$V^{(k)}=\int_{\Omega^{(k)}}\mathrm{d}\Omega=\frac{1}{4}\sum_{j=1}^{n_e^{(k)}}V_e^{(j)} \tag{9}$$