Engineering Analysis with Boundary Elements 95 (2018) 160–166

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

The precise integration method for semi-discretized equation in the dual reciprocity method to solve three-dimensional transient heat conduction problems

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ARTICLE INFO

Keywords:
Dual reciprocity method
Precise integration method
Transient heat conduction
Boundary integral equation method
Matrix exponential function

ABSTRACT

A dual reciprocity precise integration method (DRPIM) is presented to solve 3-dimensional transient heat conduction problems. In the presented method, the transient heat conduction problem, which is an initial boundary value problem, is transformed into an initial value problem (IVP) through a dual reciprocity scheme. Then an analytical solution, which is described by terms of the matrix exponential function (MEF), to this IVP is applied. A precise integration method (PIM) is finally applied to compute the MEF accurately. The accuracy and stability of the presented method are demonstrated by three numerical examples.

1. Introduction

With applying Fourier's heat conduction law, the transient heat conduction problem is usually governed by a partial differential equation (PDE) of parabolic type. The most popular numerical method for the transient heat conduction problem is the finite difference method (FDM) since it seems to be very nature in the solution to the transient heat conduction problem [1]. However, in the application of FDM for transient heat conduction problem, numerical stability is a very cumbersome problem. In order to circumvent this numerical stability problem, many criterions have been proposed to guarantee the numerical stability [2].

Besides the FDM, many other numerical methods for transient heat conduction problem have also been developed such as the finite volume method (FVM) [3], the finite element method (FEM) [4], the meshless methods(MM) [5–9] and the boundary element method(BEM) [10]. Among these methods, the BEM is very attractive since it is considered as a boundary type method which involves only boundary meshes but not domain meshes. Due to the boundary only advantage, a large quantity of computational costs could be saved. In many applications of BEM for transient heat conduction problem, the finite difference (FD) scheme is usually applied for time discretization. In those applications, the stability of the method depends on the discretization scheme. Thus, the FDBEM is also conditionally stable [11].

The BEM implementation for transient heat conduction problem can be classified into two types. In one type of BEM implementation, the time-dependent fundamental solution to unsteady heat conduction problem is applied [12–16]. While in the other type of BEM implementation, the time-independent fundamental solution to steady heat conduction problem is applied [17–22]. In the implementation with time dependent fundamental solution, there are two different approaches to deal with the affection from the front steps. The one is the time convolution method in which each step should considers results obtained in all the front steps [11–15]. This approach is very time consuming especially in the case that many time steps are involved. Many schemes have been developed accelerate this time convolution process [12–14]. Gupta et al. expanded the fundamental solution into Taylor series [12]. The time variable was separated from all the terms in the series after expansion. Making use of the monotonicity of the fundamental solution, Chatterjee and Ma et al. developed a fast time convolution algorithm to accelerate the calculation of the integrals [13,14]. The other approach is usually called by the quasi-initial condition method in which the temperature distribution all around the considered domain computed in the current time step is considered as the initial temperature distribution in the next time step. Zhou et al. applied this quasi-initial condition method for the transient heat conduction [16]. This method, however, was found to be numerically unstable. In order to circumvent this numerically unstable problem, a time step amplification method is developed.

In those implementations of quasi-initial condition method above, however, the domain integral, which is introduced by the quasi-initial temperature, are involved in each step. A domain mesh is required if we directly calculate the domain integral. Thus, the boundary only advantages of the BEM vanished. Moreover, in the case of employing small time steps, numerical computation of the domain integral becomes

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https://doi.org/10.1016/j.enganabound.2018.07.005
Received 8 May 2018; Received in revised form 15 June 2018; Accepted 23 July 2018

difficult due to the sharp variation of the fundamental solution. In order to avoid the direct computation of the domain integral, Gao and Yang introduced a radial integral method (RIM) in the BEM to transform the domain integral into the boundary integral [17–20]. Partridge et al. firstly applied the dual reciprocity boundary element method (DRBEM) for transient heat conduction [21]. Guo et al. applied a triple reciprocity method to avoid the domain integrals that appear in the boundary integral equation of the transient heat conduction problem [22].

In most of implementations of BEM for transient heat conduction problem, the finite difference (FD) scheme is usually employed to discrete the time variable. As stated before, the FDBEM is conditionally stable. For large time step cases, the numerical results become unstable. In order to circumvent this cumbersome problem, Yu et al. coupled the DRBEM and the precise time-domain expanding method for transient heat conduction and avoided the direct computation of the domain integral [23]. Yao et al. equipped the RIM and the precise integration method (PIM), which is firstly proposed by Zhong et al. to solve structure dynamic problems [24], in the BEM for transient heat conduction [25]. In the PIM, a general solution, which is described by matrix exponential function, for initial value problem is applied. Thus, it is unconditionally stable. Zhong and Yang developed a special scheme to compute the matrix exponential function precisely [26]. The PIM is considered to be a promising method for initial boundary value problems including transient heat conduction, diffusion, wave propagation and structural vibration.

This paper will present a new implementation of the DRBEM and the PIM to solve transient heat conduction problems. In the application, the transient heat conduction problem is firstly taken as a quasi-steady state problem and the derivative of the temperature respect to time is considered as the equivalent heat source. The DRM is applied to transform the domain integral about the heat source into the boundary integral. After discretization for space variables, an initial value problem about the distribution of temperature inside domain will be obtained. The PIM is finally applied to solve this initial value problem precisely. After the computation of temperatures at domain nodes, the boundary quantities including both temperatures and flux will be computed through a boundary integral equation. It should be noted that, there are two major difference in the application of DRM in this paper from the traditional application of DRM. The first one is the locations of the RBF points. In the presented method, the RBF interpolation points locate inside domain but not on the boundary. Thus, a lot of computational costs can be saved especially for problems on thin-shell like structures. The other one is the assembling of the equations. The boundary temperatures and flux are both considered to be unknowns at the first. The boundary condition is introduced in the supplement equations. Thus, the boundary condition of Robin type can be imposed naturally.

Three numerical examples concerning transient heat conduction problems on three different structures will be presented to illustrate the accuracy and the stability of the method.

This paper is arranged as follows. Some basic knowledges about DRBEM for transient heat conduction problems will be introduced in Section 2. The PIM will be described in detail in the following section. In HYPERLINK \l "sec0004" Section 4 , three validation numerical examples will be presented. This paper ends with conclusions in the last section.

2. The dual reciprocity method for transient heat conduction

The DRM is considered as an attractive method with extends the application area of the BEM to nonhomogeneous problems. In the DRM, the radial basis function (RBF) interpolation plays a great important role. The particular solution of the RBF to the corresponding problem is applied in the reciprocity process to convert the domain integral, which appears in the boundary integral equation and is related to the nonhomogeneous term, into boundary integrals.

The transient heat conduction problem in homogeneous media is usually stated by:

$$\begin{cases} k\nabla^2 u(\mathbf{x},t) + Q(\mathbf{x},t) = \rho c \dot{u}(\mathbf{x},t), & \forall \mathbf{x}\in\Omega \\ u(\mathbf{x},t) = \bar{u}(\mathbf{x},t), & \forall \mathbf{x}\in S_u \\ -k\dfrac{\partial u(\mathbf{x},t)}{\partial n(\mathbf{x})} \equiv q(\mathbf{x},t) = \bar{q}(\mathbf{x},t), & \forall \mathbf{x}\in S_q \\ q(\mathbf{x},t) = \beta(u(\mathbf{x},t) - u_a(\mathbf{x},t)), & \forall \mathbf{x}\in S_R \\ u(\mathbf{x},t_0) = u_0(\mathbf{x}), & \forall \mathbf{x}\in\Omega \end{cases} \tag{1}$$

in which $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$ is the Laplacian in 3D. u stands for the temperature, q denotes for flux rate across the boundary, dot over the temperature means variation rate of temperature along time variable t. Q is the heat source, k, $\rho$and c are heat conductivity, density and heat capacity of the material, respectively. Ω is the considered domain, n is the outward normal on the boundary. S_u, $S_q$and S_R denote for the boundary of Dirichlet type, Neumann type and Robin type, respectively. $\bar{u}$ is specified temperature on the boundary S_u. $\bar{q}$ is the specified flux on the boundary S_q . β is the heat exchange parameter on the boundary S_R. u_a is the temperature of the fluent over the boundary S_R. $u_0$stands for the known temperature at initial time t_0. The symbol dot over the variable denotes for the derivatives respect to time.

In the BEM, the governing equation in problem (1) is converted to a boundary integral equation with the help of the fundamental solution of a steady state heat conduction problem.

$$\frac{1}{\rho c}\int_\Gamma u(\mathbf{x},t)\left(k\frac{\partial u^*(\mathbf{y},\mathbf{x})}{\partial n(\mathbf{x})}\right)d\Gamma(\mathbf{x}) - \frac{1}{\rho c}\int_\Gamma u^*(\mathbf{y},\mathbf{x})\left(k\frac{\partial u(\mathbf{x},t)}{\partial n(\mathbf{x})}\right)d\Gamma(\mathbf{x}) + \frac{1}{\rho c}\int_\Omega u^*(\mathbf{y},\mathbf{x})Q(\mathbf{x},t)d\Omega(\mathbf{x}) + \frac{k}{\rho c}\int_\Omega u^*(\mathbf{y},\mathbf{x})\dot{u}(\mathbf{x},t)d\Omega(\mathbf{x}) = C(\mathbf{y})u(\mathbf{y},t) \tag{2}$$

Where

$$C(\mathbf{y}) = \begin{cases} 0 & \mathbf{y}\in\overline{\Omega\cup\Gamma} \\ 1 & \mathbf{y}\in\Omega \\ \theta & \mathbf{y}\in\Gamma \end{cases} \tag{3}$$

θ is the solid angle of the boundary, and θ=0.5 when $\mathbf{y}$ locates at a smooth boundary.

$$u^*(\mathbf{y},\mathbf{x}) = \frac{1}{4\pi r(\mathbf{y},\mathbf{x})} \tag{4}$$

is called the fundamental solution of a steady state heat conduction problem. It satisfies:

$$\nabla^2 u^*(\mathbf{y},\mathbf{x}) = -\delta(\mathbf{y},\mathbf{x}) \tag{5}$$

For convenience, we omit the heat source inside domain. Then we have the following integral equation.

$$C(\mathbf{y})u(\mathbf{y},t) = \frac{1}{\rho c}\int_\Gamma u(\mathbf{x},t)\left(k\frac{\partial u^*(\mathbf{y},\mathbf{x})}{\partial n(\mathbf{x})}\right)d\Gamma(\mathbf{x}) - \frac{1}{\rho c}\int_\Gamma u^*(\mathbf{y},\mathbf{x})\left(k\frac{\partial u(\mathbf{x},t)}{\partial n(\mathbf{x})}\right)d\Gamma(\mathbf{x}) - \frac{k}{\rho c}\int_\Omega u^*(\mathbf{y},\mathbf{x})\dot{u}(\mathbf{x},t)d\Omega(\mathbf{x}) \tag{6}$$

The DRM is applied to convert the domain integral into boundary integral. We applied a RBF interpolation for $\dot{u}(\mathbf{y},t)$.

$$\dot{u}(\mathbf{y},t) = \sum_i \alpha_i(t)\varphi_i(\|\mathbf{y}-\mathbf{z}_i\|) \tag{7}$$

In which $\phi_i(\|\mathbf{y}-\mathbf{z}_i\|)$ is the RBF centered at $\mathbf{z}_i$. $\|\bullet\|$ denotes the Euclid norm in 3D space. Although there are many kinds of RBF, we employ the multiquadric function in this paper.

$$\varphi_i(\|\mathbf{y}-\mathbf{z}_i\|) = \sqrt{r^2+s^2} \tag{8}$$

s is the shape parameter of the multiquadric function. Nd centers of RBF locate inside the domain uniformly or randomly. Then after collocation