Engineering Analysis with Boundary Elements 95 (2018) 175–186

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Generalized finite difference method for solving the double-diffusive natural convection in fluid-saturated porous media

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ARTICLE INFO

Keywords:
Meshless numerical scheme
Double-diffusive natural convection
Generalized finite different method
Newton–Raphson method
Porous media

ABSTRACT

In this paper, the generalized finite difference method (GFDM) combined with the Newton–Raphson method is proposed to accurately and efficiently simulate the steady-state double-diffusive natural convection in parallelogrammic enclosures filled with fluid-saturated porous media. The natural convection in fluid-saturated porous media, which is interesting in regard to the heat-transferring range, involves different physical compositions to affect the fluid flow. For the mathematical formulations of the natural convention, the governing equations are a system of highly-nonlinear partial differential equations, so the approximate solutions for the natural convention mainly depend on a suitable numerical scheme. In this study, the GFDM, a newly-developed meshless method, is adopted for the spatial discretization of the non-linear governing equations, since it can avoid setting up the mesh in the computational domain and implementing the numerical quadrature. The localization of the GFDM will result in a sparse system, while the derivatives at each node can be expressed as linear combinations of nearby function values with different weighting coefficients. After a system of nonlinear algebraic equations is yielded by the spatial discretization of the GFDM, the two-steps Newton–Raphson method is adopted to efficiently solve this resultant sparse system owing to the localization of the GFDM. Three numerical examples are presented to demonstrate the applicability and stability of the proposed meshless numerical scheme. Besides, the numerical results are compared with other solutions to show the accuracy of the proposed method.

1. Introduction

The double-diffusive natural convection in parallelogrammic enclosures filled with fluid-saturated porous media is driven by the variation of the densities affected by various physical components, such as temperature and concentration. To analyze this problem by using any mathematical method is extremely difficult, since the governing equations are highly non-linear [1,2]. Hence, it is necessary to adopt an appropriate numerical scheme in order to accurately and efficiently deal with the double-diffusive natural convection in fluid-saturated porous media. The review of the double-diffusive natural convection in fluid-saturated porous media has been provided by Nield and Bejan [1]. In the past, the double-diffusive natural convection in fluid-saturated porous media has been numerically studied by some researchers [2–6]. For example, in 2004, Costa [2] used the finite element method to solve the two-dimensional double-diffusive natural convection in parallelogrammic enclosures filled with fluid-saturated porous medium. Bourich et al. [3] applied the alternating direction implicit method to simulate the time-dependent double-diffusive natural convection in a square enclosure. Kramer et al. [4] adopted the Navier–Stokes equations to describe the heat and mass transfer, and applied the boundary domain integral method to simulate the double-diffusive natural convection. In addition, Stajnko et al. [5] used the boundary element method to analyze the three-dimensional double-diffusive natural convection in porous media. Recently, Fan et al. [6] utilized the local radial basis function (RBF) collocation method (LRBFCM), one of the most-promising meshless methods, to solve the double-diffusive natural convection in parallelogrammic enclosures filled with fluid-saturated porous media. Although the LRBFCM can efficiently and accurately analyze the double-diffusive natural convection, the shape parameter in the RBF, which will have great influence on the accuracy of numerical results, should be determined by trial-and-error tests. The optimal choice of the shape parameter in the RBF is still an open question. According to the above discussions, it can be found that many numerical schemes have been adopted to accurately analyze the double-diffusive natural convection. Most of these numerical schemes are belonged to the classical mesh-dependent methods, which required time-consuming tasks of mesh generation and numerical quadrature. Thus, in this paper, we proposed an efficient and stable meshless numerical scheme to

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https://doi.org/10.1016/j.enganabound.2018.06.014
Received 8 February 2018; Received in revised form 21 June 2018; Accepted 21 June 2018

investigate the two-dimensional steady-state double-diffusive natural convection in parallelogrammic enclosures filled with fluid-saturated porous media.

From the viewpoint of requirement of mesh or randomly-distributed nodes, the numerical methods for spatial discretizations of partial differential equations (PDEs) can be generally sorted to two families: the mesh-dependent methods and the meshless methods. In order to simply the numerical procedures and accelerate the computer simulation, to use the meshless methods can avoid the time and labor to generate mesh and to implement numerical quadrature. Thus, there are various meshless methods proposed in the past two decades, such as the method of fundamental solutions [7,8], the boundary knot method [9], the Trefftz method [10,11], the singular boundary method [12–14], the boundary particle method [15], the method of particular solutions [16,17], the LRBFCM [6], the meshless local Petrov–Galerkin method [18] and the generalized finite difference method (GFDM) [19-33], etc. Among them, the GFDM is one of the most-promising domain-type meshless methods.

The GFDM is evolved from the classical finite difference method, which is a simple and accurate numerical method. The numerical procedures of the GFDM are developed from the Taylor series expansion and the moving-least-squares method, so the derivatives with respect to space coordinates can be expressed as linear combinations of nearby function values within a star. By comparing with the finite difference method, the GFDM can use non-uniform grids and be easily applied to problems in irregular computational domains. In 2001, Benito et al. [19] proposed the explicit formulations of the GFDM and tested several factors of the GFDM [20]. Then, GFDM was applied to parabolic and hyperbolic PDEs [21] and advection–diffusion equation [22]. After these initial works of the GFDM mentioned above, researchers paid attention to the GFDM and applied the GFDM to deal with some engineering problems, such as, two-dimensional nonlinear obstacle problems [23], density-driven groundwater flows [24], thermoelastic wave propagation analysis [25], non-Fickian diffusion–elasticity analysis [26], two-dimensional shallow water equations [27], inverse heat source problems [28], non-linear elliptic PDEs [29], wave problems [30], three-dimensional inhomogeneous Helmholtz-type equations [31], coupled thermoelastic analysis [32], etc. From the above discussions of the GFDM, it is evident that the GFDM has been adopted for numerical solutions of complicated PDEs and has great potential to be utilized to deal with various engineering problems. Accordingly, we adopted the GFDM for spatial discretization of the governing equations of two-dimensional double-diffusive natural convection in parallelogrammic enclosures filled with fluid-saturated porous media in this paper.

While the GFDM is responsible for the spatial discretization of the governing equation of the double-diffusive natural convection, considered in this paper, a system of nonlinear algebraic equations (NAEs) will be yielded. Therefore, an efficient and stable solver for NAEs is required. In the past, many solvers for NAEs have been developed to efficiently solve the system of NAEs, such as the Newton–Raphson method [34], the exponentially-convergent scalar homotopy algorithm (ECSHA) [6,35], the FTIM [33,36], etc. Although the ECSHA and the FTIM can avoid the time-consuming calculations of the inverse of the Jacobian matrix [6,34–36], there are some troublesome free parameters, which will greatly influence on the accuracy and efficiency of computer simulation, in these two solvers. In order to achieve highly-efficient solver for a system of NAEs, our research group [24] in 2014 re-formulated the iterative process of the Newton–Raphson method into two sequential steps so as to avoid the time-consuming calculation of the inverse of the Jacobian matrix. Within one iteration step, a sparse system of linear algebraic equations, formed by the GFDM, should be efficiently solved, and then the new physical value at the next iteration step can be updated. The two sequential steps should be repeatedly implemented until the convergent solutions are acquired. Using these two sequential steps of the Newton–Raphson method not only can avoid computing the inverse of Jacobian matrix but also keep the great efficiency of the Newton–Raphson method, especially for large-scale engineering problems.

The study in this paper might be regarded as an extension of our previous research [24] to steady-state double-diffusive natural convection in parallelogrammic enclosures filled with fluid-saturated porous media. In [24], we combined the GFDM and the two-steps Newton–Raphson method to accurately study the time-dependent natural convection in groundwater and the interaction between the streamfunction and concentration are efficiently simulated by the proposed method. In comparing with [24], in this paper we adopted the GFDM and the Newton–Raphson method to accurately solve more complicated system of PDEs, which involves the interaction between streamfunction, temperature and concentration. Thus, there are three nonlinear PDEs with three physical variables in the present study, which pose a great challenge to numerical simulation. In addition, the initial guess of Newton–Raphson method in a time-dependent problem can be easily determined by using the numerical results from the previous time step [24]. On the contrary, it is a non-trivial task to study the initial guess of Newton–Raphson method in the steady-state problem in this paper, since there is no solution in the previous time step can be used. In addition to the differences of mathematical formulation and initial guess of iteration between this study and [24], in this paper we used the parallelogrammic computational domain and arbitrarily-distributed nodes, which does not appear in [24], to verify the features of proposed meshless scheme. Therefore, the combination of the GFDM and the two-step Newton–Raphson method is proposed in this paper to accurately and efficiently study the double-diffusive natural convection in parallelogrammic enclosures filled with fluid-saturated porous media. Besides, different parameters, such as the number of total nodes, the number of nodes in a star, the initial guess of iteration and the buoyancy ratio, are adopted to demonstrate the merits of proposed numerical scheme. The successful study in this paper can lead the extension of the proposed numerical scheme to some challenging engineering applications of steady-state nonlinear multi-physics problems in the future.

This paper is organized as follows: the motivation of this study and the discussions of relevant research are provided in the first section. The mathematical formulation of the double-diffusive natural convection in fluid-saturated porous media is described in the second section. Then, the proposed meshless numerical schemes are described in Section 3. In Section 4, we presented three numerical examples to verify the accuracy, efficiency and stability of proposed numerical scheme, while the conclusions and discussions, based on the numerical results and comparisons, are drawn in the final section.

2. Mathematical formulation of the physic problem

Considering the pressure–velocity link for the fluid in the porous medium given by the Darcy's law and defining the streamfunction $\psi(x,y)$ through its 1st-order derivatives, the dimensionless governing equations can be written as [1,2]:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + Ra\frac{\partial}{\partial x}(T + NC) = 0, (x, y) \in \Omega, \tag{1}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = 0, (x, y) \in \Omega, \tag{2}$$

$$\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} - Le\frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x} + Le\frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y} = 0, (x, y) \in \Omega, \tag{3}$$

where (x, y) is the spatial coordinates, $T(x, y)$ is the dimensionless temperature, $C(x, y)$ is the dimensionless concentration, Ra is the Rayleigh number, Le is the Lewis number, N is the buoyancy ratio and Ω is the computational domain. The detailed discussions of the above equations can be found in some previous studies [1,2]. Since ψ, T and C are the unknown variables, so the multiplicative terms of the first derivative are the nonlinearity in Eqs. (2) and (3). Therefore, those nonlinear terms