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Comparative study of time and frequency domain approaches in contact problem for the I-mode crack under harmonic loading

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ABSTRACT

In this paper two approaches, time-domain (TD) and frequency domain (FD) have been applied for the solution of the dynamic unilateral contact problem. We investigate crack in two-dimensional (2-D) plane, homogeneous, isotropic and linear elastic solids under action of the longitudinal P-wave with considering unilateral contact of the crack edges. For the problem solution a special iterative algorithm of the Udzava's type and boundary element methods (BEM) in the time-domain (TD) and frequency domain (FD) are used. For evaluation of the hypersingular integrals involved in BIEs a special regularization process that converts these integrals to regular ones is applied. Simple regular formulas for their calculation are presented. Numerical results for the crack opening, contact forces and dynamic stress intensity factor (SIF) for the case of a finite I-mode crack in an infinite 2-D domain which is subjected to a harmonic loading with considering crack edges unilateral contact interaction are presented and discussed.

1. Introduction

Mechanical contact is one of the most common and important solid bodies interactions. Contact and friction are phenomena that are of extreme importance in uncountable scientific and engineering applications Popov [43]. In many cases dynamic effects play an important role in such interaction Brogliato [5]. Contact problems are inherently nonlinear, since the actual surface on which these bodies meet is generally unknown a priori and must be determined as part of the solution. There are many approaches and algorithms for solution of the unilateral contact problems with friction Meguid and Czekanski [40], Wriggers [48]. Modern approach for such problems solution consists of the application of variational inequalities and nonsmooth analysis Brogliato [5], Panagiotopolos [42]. Because contact problems are strongly nonlinear, they usually are solved numerically, using finite element methods (FEM) Antes and Panagiotopoulos [4], Kikuchi and Oden [36] and boundary element methods (BEM) Antes and Panagiotopoulos [4], Man [38].

In the case of dynamical contact problems, the traditional method to find their solution is the TD approach, when the problem is solved in a real time domain using time various step methods. Such approach with exploration of the FEM, in particular, has been developed in our publications. In other publications Czekanski and Meguid [9,10], Czekanski et al. [12] researchers have been in the development of novel quasi-static and dynamic frictional contact formulations, using a mathematically rigorous variational inequalities approach and new solution techniques in the form of non-differential optimization. A new time-integration scheme for dynamic contact problems, in which energy and momentum are conserved during impact has been developed in Czekanski and Meguid [11]. The accuracy and robustness of this approach resulted in its being adopted by the major commercial Finite Element Analysis software "ABAQUS" and the above mentioned paper is referenced in its theory manual. For further references related with the TD FEM approach, one can see Brogliato [5], El-Abbasi et al. [18], Meguid and Czekanski [40], Eck et al. [16], Oden and Martins [41], Panagiotopolos [42].

The TD approach with exploration of the BEM has been developed in numerous publications mostly without application to the contact problems. Among others, we have to mention here Gallego and Dominguez [24], Israil and Banerjee [33,34], Wen et al. [47], Garcia-Sanchez et al. [25,26,28], Zhang [49] where isotropic and anisotropic solids have been considered. The TD approach with exploration of the BEM has been applied for solution of the dynamic unilateral contact problem in Hirose [20] and contact problem with friction in Stavroulakis et al. [46].

Except for the TD, there are two different BEM formulations, namely, the frequency-domain Denda et al. [13], Dineva et al. [14] and the Laplace domain Fedelinski et al. [22], which are often applied to the elastodynamic crack analysis. Comparative study of the different BEM formulations and analysis of their accuracy and efficiency is performed in Chirino et al. [8], Garcia-Sanchez and Zhang [27], Manolis [39] for elastodynamic problems without considering possibilities for contact integration. For more information and further references related to

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application of the BEM for dynamical problems please see Aliabadi [3], Dominguez [15].

It is important to mention that fracture dynamics problems are usually solved without taking into account the possibility of the contact interaction of opposite crack edges Freund [23], Sih [44]. Analysis of static fracture mechanics problems demonstrates that taking the crack edge contact interaction into account may significantly affect the fracture mechanics criteria. In dynamic problems, the effects of the crack edge contact interaction can significantly exceed those in the static case. Moreover, in dynamic problems it is very difficult to find classes of loads which do not cause crack edge contact interaction. The importance of the influence of crack edges contact interaction on fracture mechanics criteria has been investigated and discussed in Guz and Zozulya [30–32].

Formulation of the elastodynamic problem for a cracked body, that takes into account the possibility of crack edge contact interaction and the formation of areas with close contact, adhesion and sliding, was reported first time in Zozulya [51]. In addition, an algorithm for the problem solution with considering crack edges unilateral contact interaction and friction was elaborated and adapted for the case of harmonic load in Zozulya [52]. The algorithm is based on a theory of subdifferentional functionals and finding of their saddle points. Such algorithms are usually called Uzava's type Cea [6], Ekeland and Temam [17]. Mathematical aspects of the problem and algorithm convergence were investigated in Zozulya [53]. It has been shown that the algorithm may be considered as a compressive operator, acting on special Sobolev's functional space. Briefly speaking the algorithm comprises two parts. The first part is a solution to an elastodynamic problem for bodies with cracks, without taking into account contact conditions. The second part is a projection of the founded solution on the set of unilateral contact restrictions and friction.

In Zozulya [54] a comparative study of the TD and FD BEM approaches for solution of the dynamic frictional contact problem in the case of the III-mode crack interacting with harmonic shear SH-wave. Based on authors knowledge, there is no analysis dedicated to the problems of the I-mode crack interacting with harmonic longitudinal P-wave that takes into account unilateral contact interaction.

In this paper, a comparative study of the TD and the FD BEM formulations, analysis of their accuracy and efficiency is performed for the case of unilateral contact problem for plane I-mode crack interacted with harmonic longitudinal P-waves. Numerical examples for computing the crack opening, the contact forces and the dynamic SIFs are presented to compare the accuracy and the efficiency of the two different BEM formulations.

2. Formulation of the problem

Let's a homogeneous linearly elastic body occupies in an Euclidean space volume V. Its boundary ∂V is piece-wise smooth and consists of sections ∂V_p and ∂V_u, to which the values of vectors of surface load $\boldsymbol{\psi}(\mathbf{x}, t)$ and displacements $\boldsymbol{\phi}(\mathbf{x}, t)$ respectively, are assigned. It is well known (see Achenbach [2], Eringen and Suhubi [19] that if the stress-strain state of elastic body depends on only two coordinates $\mathbf{x}_\alpha = (x_1, x_2) \in V \subset R^2$ and time $t \in \mathfrak{I}$, then the main equations of elastodynamics are decomposed into two independent parts: plane and antiplane problems. Following Achenbach [2], Eringen and Suhubi [19] we consider here plane equations of elastodynamics. The elastic body may be subjected to volume $\mathbf{b}(\mathbf{x}_\alpha,t) = b_\alpha(\mathbf{x}_\alpha,t)\mathbf{e}_\alpha$ and surface $\mathbf{p}(\mathbf{x}_\alpha,t) = p_\alpha(\mathbf{x}_\alpha,t)\mathbf{e}_\alpha$ forces. The stress–strain state of the classical elastic continua is defined in terms of the displacements vector $\mathbf{u}(\mathbf{x}_\alpha,t) = u_\alpha(\mathbf{x}_\alpha,t)\mathbf{e}_\alpha$, the stress $\boldsymbol{\sigma}(\mathbf{x}_\alpha,t) = \sigma_{\alpha\beta}(\mathbf{x}_\alpha,t)\mathbf{e}_\alpha \otimes \mathbf{e}_\beta$ and strain $\boldsymbol{\varepsilon}(\mathbf{x}_\alpha,t) = \varepsilon_{\alpha\beta}(\mathbf{x}_\alpha,t)\mathbf{e}_\alpha \otimes \mathbf{e}_\beta$ tensors, which have the form

$$\boldsymbol{\sigma} = \begin{vmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{vmatrix}, \ \boldsymbol{\varepsilon} = \begin{vmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{22} \end{vmatrix}, \ \mathbf{u} = \begin{vmatrix} u_1 \\ u_2 \end{vmatrix}, \ \mathbf{b} = \begin{vmatrix} b_1 \\ b_2 \end{vmatrix}, \ \mathbf{p} = \begin{vmatrix} p_1 \\ p_2 \end{vmatrix} \tag{1}$$

We consider here the possibility for a crack edges contact interaction during the deformation. The contact forces $\mathbf{q}(\mathbf{x}_\alpha,t) = q_\alpha(\mathbf{x}_\alpha,t)\mathbf{e}_\alpha$ which arise on the cracks edges during the interaction are denoted by

$$\mathbf{q}(\mathbf{x}_\alpha, t) = -\boldsymbol{\sigma}(\mathbf{x}_\alpha, t) \cdot \mathbf{n}(\mathbf{x}_\alpha) \tag{2}$$

where $\mathbf{n}(\mathbf{x}_\alpha) = n_\alpha(\mathbf{x}_\alpha)\mathbf{e}_\alpha$, is a unit vector normal to the crack surfaces, $n_\alpha(\mathbf{x}_\alpha) = n_\alpha^+(\mathbf{x}_\alpha)$ for $\mathbf{x} \in \Omega^+$ and $n_\alpha(\mathbf{x}_\alpha) = -n_\alpha^-(\mathbf{x}_\alpha)$ for $\mathbf{x}_\alpha \in \Omega^-$ $n_\alpha^+(\mathbf{x}_\alpha)$ and $n_\alpha^-(\mathbf{x}_\alpha)$ are directed to the positive side of the opposite cracks edges.

The mutual displacements of the cracks edges can be described by the displacement discontinuity vector, which is presented in the form

$$\Delta\mathbf{u}(\mathbf{x}_\alpha, t) = \mathbf{u}^+(\mathbf{x}_\alpha, t) - \mathbf{u}^-(\mathbf{x}_\alpha, t) \tag{3}$$

where $\mathbf{u}^+(\mathbf{x}, t)$ and $\mathbf{u}^-(\mathbf{x}, t)$ are displacements of the crack opposite edges.

On the crack edges, the vectors of contact forces and displacement discontinuity must satisfy unilateral contact constraints with friction. In Antes and Panagiotopoulos [4], Guz and Zozulya [30], Panagiotopoulos [42] it was shown that unilateral boundary conditions with friction have the form

$$\begin{aligned} &\Delta u_n \geq -h_0, \quad q_n \geq 0, \quad (\Delta u_n + h_o) q_n = 0 \\ &|\mathbf{q}_\tau| \leq k_\tau q_n \Rightarrow \partial_t \Delta\mathbf{u}_\tau = 0, \\ &|\mathbf{q}_\tau| = k_\tau q_n \Rightarrow \partial_t \Delta\mathbf{u}_\tau = -\lambda_t \mathbf{q}_\tau, \forall \mathbf{x} \in \Omega^+ \cup \Omega^-, \forall t \in \mathfrak{I} = [t_0, t_1] \end{aligned} \tag{4}$$

where q_n, $\mathbf{q}_\tau$ and Δu_n, $\Delta\mathbf{u}_\tau$ are the normal and tangential components of the vectors of contact forces and displacement discontinuities, respectively, h_0 is the initial crack opening, and k_τ and λ_τ are coefficients depending on the properties of the contacting surfaces Ω^+ and Ω^-.

Following Brogliato [5], Panagiotopoulos [42], Kanno [35] we rewrite unilateral contact conditions with friction (4) in terms of nonsmooth analysis. For this purpose, we introduce corresponding nonsmooth potentials, which will be called superpotentials.

For the first unilateral contact condition in (4) the corresponding superpotential has the form

$$j_n(\Delta u_n(\mathbf{x}_\alpha, t)) = \begin{cases} 0, & \text{if } \Delta u_n(\mathbf{x}_\alpha, t) \geq h_0 \\ \infty, & \text{otherwise} \end{cases} \tag{5}$$

For the second contact condition with friction in (4) the corresponding superpotential has the form

$$j_\tau(\partial_t \Delta\mathbf{u}_\tau(\mathbf{x}_\alpha, t)) = k_\tau |q_n| |\partial_t \Delta\mathbf{u}_\tau(\mathbf{x}_\alpha, t)| \tag{6}$$

We will obtain all of the equations of linear elastodynamics including contact conditions (4) for the cracked body by considering the generalized variational principle Gurtin [29]. For this purpose, let us introduce the generalized functional, that depends on the functions $\mathbf{u}$, $\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$, $\Delta\mathbf{u}$ in the form

$$\begin{aligned} \Phi = &\int_{t_0}^{t_1} \left(\int_V \left(\boldsymbol{\sigma} \cdot \cdot (\nabla \cdot \mathbf{u} - \boldsymbol{\varepsilon}) + U(\boldsymbol{\varepsilon}) - \mathbf{b} \cdot \mathbf{u} + \frac{\rho}{2} \partial_t \mathbf{u} \cdot \partial_t \mathbf{u} \right) dV \right. \\ &\left. - \int_{\partial V_p} \mathbf{p} \cdot \mathbf{u} dS - \int_{\partial V_u} \boldsymbol{\sigma} \cdot \mathbf{n} \cdot (\mathbf{u} - \boldsymbol{\varphi}) dS \right) dt + \Phi(\Delta\mathbf{u}) \end{aligned} \tag{7}$$

Here $\Phi(\Delta\mathbf{u})$ is a nonsmooth potential obtained from the superpotentials (5) and (6) in the form

$$\Phi(\Delta\mathbf{u}) = \int_{t_0}^{t_1} \left(\int_\Omega j_n(\Delta u_n(\mathbf{x}_\alpha, t)) dS + \int_\Omega j_\tau(\partial_t \Delta\mathbf{u}_\tau(\mathbf{x}_\alpha, t)) dS \right) dt \tag{8}$$

The superpotentials (5) and (6) are defined locally, at every point and the nonsmooth potential (8) globally, in a generalized sense. For details refer to Panagiotopoulos [42], Zozulya [53].

Let us consider a variation of the functional (7) by taking into account that all of above mentioned functions are independent. After some transformations and simplifications, we will get

$$\begin{aligned} \delta\Phi = &\int_{t_0}^{t_1} \left(\int_V \left(\delta\boldsymbol{\sigma} \cdot \cdot (\nabla \cdot \mathbf{u} - \boldsymbol{\varepsilon}) - (\nabla \cdot \boldsymbol{\sigma} + \mathbf{b} - \rho \partial_t^2 \mathbf{u}) \cdot \delta\mathbf{u} + \left(\frac{\partial U}{\partial \boldsymbol{\varepsilon}} - \boldsymbol{\sigma} \right) \delta\boldsymbol{\varepsilon} \right) dV \right. \\ &\left. + \int_{\partial V_p} (\boldsymbol{\sigma} \cdot \mathbf{n} - \mathbf{p}) \cdot \delta\mathbf{u} \cdot dS - \int_{\partial V_u} \delta\boldsymbol{\sigma} \cdot \mathbf{n} \cdot (\mathbf{u} - \boldsymbol{\varphi}) dS \right) dt + \delta\Phi(\Delta\mathbf{u}) = 0 \end{aligned} \tag{9}$$