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Complex Fourier element shape functions for analysis of 2D static and transient dynamic problems using dual reciprocity boundary element method

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ABSTRACT

In this paper, the boundary element method is reformulated using new complex Fourier shape functions for solving two-dimensional (2D) elastostatic and dynamic problems. For approximating the geometry of boundaries and the state variables (displacements and tractions) of Navier's differential equation, the dual reciprocity (DR) boundary element method (BEM) is reconsidered by employing complex Fourier shape functions. After enriching a class of radial basis functions (RBFs), called complex Fourier RBFs, the interpolation functions of a complex Fourier boundary element framework are derived. To do so, polynomial terms are added to the functional expansion that only employs complex Fourier RBF in the approximation. In addition to polynomial function fields, the participation of exponential and trigonometric ones has also increased robustness and efficiency in the interpolation. Another interesting feature is that no Runge phenomenon happens in equispaced complex Fourier macroelements, unlike equispaced classic Lagrange ones. In the end, several numerical examples are solved to illustrate the efficiency and accuracy of the suggested complex Fourier shape functions and in comparison with the classic Lagrange ones, the proposed shape functions result in much more accurate and stable outcomes.

1. Introduction

Elastostatics and dynamics contain an extensive range of phenomena in engineering and physical problems including force equilibrium of special structures and analysis of structures subjected to earthquake, vibratory motor, collision and explosion loads. In these cases, the wave propagation is expressed by a governing linear partial differential equation associated with suitable initial and boundary conditions. In general, obtaining the solution of elastostatic and dynamic problems for the sake of analysis and design can be difficult and laborious when analytical approaches are used. Moreover, it may even become impossible when a little complexity happens in boundary conditions. Therefore, it seems reasonable in most practical engineering cases to solve them numerically. Among the numerical methods considered significantly by researchers, boundary element method (BEM) [1,2] can be mentioned. As it is obvious from its name, only the boundary needs to be discretized in this method, not the domain. Thus, fewer unknown parameters need to be stored and less computational cost and storage space will be spent. For problems such as stress concentration or infinite domains, BEM can be applied to achieve better accuracy in comparison with finite element method (FEM). Many usages of BEM in solving problems related to buckling, optimization, crack, wave propagation, and etc. are reported in the literature. [3–11]

According to the literature, three formulations known as the Laplace transform, the time domain (TD), and the domain integral techniques exist for solving elastodynamic problems with BEM [1,2]. However, the first two methods are with mathematical complexities and the third one requires domain integration, which are some challenges to be faced in these methods. In the work of Nardini and Brebbia [12–14], the well-known dual reciprocity method (DRM) was introduced to overcome these problems. With the introduction of DRM, a significant development happened in the BEM analysis of time-dependent problems. One of the advantages of DRM is benefiting from less computational cost than other methods (like TDM) due to its ability in using the simple Green's function of elastostatics for analyzing elastodynamic problems.

Various usages of the DRM for solving a broad range of problems are reported in the literature. In the works of Dehghan et al. [15–19], this method was implemented for solving various equations and problems including stochastic partial differential equations, linear Helmholtz and semi linear Poisson's equations, and etc. The DRM was applied in the solution of free and force vibration problems by Rashed et al. [20–24]. Hamzehei Javaran et al. [25–30] applied DRM for the analysis of prob-

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lems such as elastodynamic, potential and etc. For other sample usages, see Refs. [31–39].

In this study, BEM is developed for solving 2D elastostatic and dynamic problems using new class of shape functions derived from complex Fourier radial basis function (RBF). In general, two types of RBF are reported in the literature: oscillatory and non-oscillatory. For example, the conical functions [31,32], the thin plate splines [33–35], the Gaussian functions [20], multiquadrics [23,24,35], inverse multiquadric [29], and compact supported functions [21,22,37,38] are non-oscillatory RBFs, while real and complex Fourier [25–27], J-Bessel [28] and spherical Hankel RBF [30] are oscillatory ones. In this paper, an element based interpolation function is proposed by enriching an oscillatory class of RBFs based on complex Fourier functions in natural coordinates. The relation of suggested shape function corresponding to a complex Fourier element with arbitrary number of nodes n is explicitly derived. These shape functions are able to satisfy exponential and trigonometric function fields as well as polynomial ones. Moreover, no Runge phenomenon happens in the complex Fourier macroelements. For showing the high robustness of the proposed shape functions in modeling non-smooth boundaries, a boundary with four non-differentiable points is reproduced by a four-node complex Fourier element. For evaluating the efficiency of the present shape functions, their results are compared with analytical and classic Lagrange shape functions as well as other numerical methods reported in the literature through several numerical examples. It can be understood from their results that the proposed method is more accurate and stable using fewer degrees of freedom, and finally, less computational cost.

2. Dual reciprocity method formulation

The equation of DRM formulation for dynamic analysis of a general 2D body with a domain Ω and a boundary Γ, where $\boldsymbol{x}$ and ξ respectively represent the field and source points, can be derived as below (for more details see Refs. [1,2,12–14]):

$$c_{lk}(\xi)u_k(\xi) + \int_\Gamma p^*_{lk}u_k\,d\Gamma = \int_\Gamma u^*_{lk}p_k\,d\Gamma + \sum_{m=1}^{M}\left[c_{lk}(\xi)\Psi^m_{kj} + \int_\Gamma p^*_{lk}\Psi^m_{kj}\,d\Gamma - \int_\Gamma u^*_{lk}\eta^m_{kj}\,d\Gamma\right]\alpha^m_j \tag{1}$$

in which, u_k and p_k denote the displacement and traction fields, respectively. Moreover, u^*_{lk} and p^*_{lk} are the fundamental solutions of displacements and tractions [1]. In addition, c_{lk} represents the jump terms, and l and k are indices. Furthermore, ψ^m_{jl} and η^m_{jl} respectively indicate the fictitious displacement and traction fields derived from the concept of particular solutions for an infinite domain without boundary conditions employing a set of coefficients α^m_j and a class of RBFs.

Now, the numerical solution of the boundary integral equation (Eq. (1)) with no domain integration is accessible. In the following section, a new approximate approach consisting of present shape functions and therefore, new boundary elements are proposed.

3. Basis of construction of complex Fourier elements

The DRM is considered as one of the branches of the method of particular solutions (MPS). In other words, in the approximation of inertia term in elastodynamic problems, MPS is known as DRM. In general, the MPS is useful in solving non-homogeneous partial differential equations, $Lu = f$. Its main idea is the expansion of non-homogeneous term f by its values in interpolation nodes so that a particular solution can be obtained [27], which can be done by radial basic functions (RBFs) approximation. Thus, the governing equation is reduced to a homogeneous one. According to the literature about the process of interpolation by RBFs, no connectivity exists between interpolation nodes as elements. Now this idea comes to mind that is it possible to interpolate the state variables by using BEM that uses elements for interpolation and benefit from the advantages of RBFs as well?

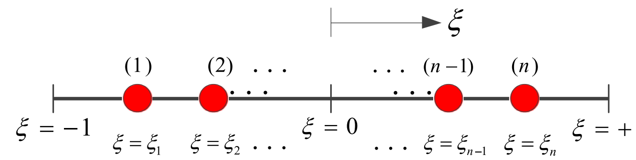

Fig. 1. A n-node boundary element with arbitrary coordinates $\xi_1,\xi_2,\ldots,\xi_n$in the natural coordinate system.

The complex Fourier functions have been already implemented by the authors [27] as RBF for the interpolation of non-homogenous terms of Navier's equation (inertia term, not the state variables) and as shape function in an element-based framework with only 3-node elements for the approximation of state variables of potential problems (potentials and fluxes) with remarkable outcomes [26]. According to the oscillating and decaying features of these functions, a good agreement between them and the nature of dynamic waves in elastodynamic problems may be established. This fact can be easily seen in the numerical results of Section 6. Here, it is tried to use complex Fourier in an element-based framework with arbitrary n-node elements for the approximation of the state variables of Navier's equation (displacements and tractions). Therefore, the idea of enriching complex Fourier RBF in the natural coordinates mapping with a number of arbitrary nodes is developed in this research.

3.1. Enrichment of complex Fourier RBF

This section explains the enrichment steps of the desired RBF for a n-node boundary element with arbitrary coordinates $\xi_1,\xi_2,\ldots,\xi_n$. To do so, polynomial terms are joined to the functional expansion that only employs RBF in the approximation

$$u_h(x) = \sum_{i=1}^{n} R_i(r)a_i + \sum_{j=1}^{m} P_j(\boldsymbol{x})b_j = \boldsymbol{R}^T(r)\boldsymbol{a} + \boldsymbol{P}^T(\boldsymbol{x})\boldsymbol{b} \tag{2}$$

in which, n and m denote the number of nodes and basis polynomial terms, respectively. Furthermore, r and $\boldsymbol{x}$ indicate the Euclidean norm among data points, and coordinates of data points, respectively. The definition of other parameters is available in Ref. [26]. Moreover,

$$\boldsymbol{\Phi}(\boldsymbol{x}) = \boldsymbol{R}^T(r)\boldsymbol{S}_a + \boldsymbol{P}^T(\boldsymbol{x})\boldsymbol{S}_b \tag{3}$$

where,

$$\boldsymbol{S}_b = \left[\boldsymbol{P}^T_m\boldsymbol{R}^{-1}_Q\boldsymbol{P}_m\right]^{-1}\boldsymbol{P}^T_m\boldsymbol{R}^{-1}_Q,\quad \boldsymbol{S}_a = \boldsymbol{R}^{-1}_Q - \boldsymbol{R}^{-1}_Q\boldsymbol{P}_m\boldsymbol{S}_b \tag{4}$$

Now, it is tried to employ the aforementioned approach in a n-node element in a natural coordinate system ξ, to be used in BEM (Fig. 1). The desired RBF is proposed as the equation below [27]:

$$R(r) = \alpha e^{i\omega r} \tag{5}$$

where, α and ω denote some constants (the so-called shape parameters) that may be chosen to increase the accuracy [27] and $e^{i\omega r} = \cos(\omega r) + i\sin(\omega r)$ represents the complex-real exponential function. The $\boldsymbol{R}(r)$ and $\boldsymbol{P}(\xi)$ vectors for the boundary element illustrated in Fig. 1 are represented as below:

$$\boldsymbol{R}(r) = \alpha\begin{bmatrix} e^{i\omega|\xi-\xi_1|} \\ e^{i\omega|\xi-\xi_2|} \\ \cdot \\ \cdot \\ \cdot \\ e^{i\omega|\xi-\xi_n|} \end{bmatrix} \tag{6}$$

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