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Analysis of thermal effect around an underground storage cavern with a combined three-dimensional indirect boundary element method

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A B S T R A C T

This paper combines a displacement discontinuity method (DDM) for elasticity and an indirect boundary element method (IBEM) for heat conduction to investigate three dimensional thermal displacements and stresses, such as those around an underground storage cavern. The thermal displacements and stresses due to temperature change are expressed in terms of a thermal displacement potential function. For a constant point heat source, we present a derivation for solution of the thermal displacement potential function and the solution is the same as found in literature. A hybrid scheme with semi-analytical integration and a direct numerical integration are used to integrate the thermal displacement potential function due to a constant heat source on a triangular region. The hybrid scheme is used to overcome a difficulty arising with the semi-analytical integration for points whose projection on the integration plane is close to the vertices of the triangle. The combined IBEM with the hybrid integration scheme is first verified with the analytical solution for an infinite elastic body with a spherical cavity. The results agree well, although differences are observed in the early stages of simulation. Then the method is used to simulate the deformation around a liquefied natural gas underground storage cavern.

1. Introduction

Temperature changes occur constantly in any non-controlled environments. In any deformable medium, changes in the surrounding temperature affect deformation and stress distribution. The deformation of some materials is more sensitive to temperature changes than in others, and in some situations the temperature variation can be very large. The possible effects of temperature change must therefore be considered in any applications with such conditions.

Thermal stress due to temperature change must be considered in many engineering applications, such as mechanical engineering, where friction between two parts of a machine could change the temperature. In turn, this alters the deformation of the parts, affecting the machine's operation. Electronic devices may be very sensitive to temperature change, because current flow can affect the temperature, causing deformation. Thermal stress has large effects in many civil and geotechnical engineering applications, particularly in underground energy storage facilities. In geothermal energy engineering [1,2], where underground heat is transferred by flowing water to the ground's surface, temperature change affects the rock deformation. Temperature changes can be due to heat release, which increases temperatures, as well as heat absorption, which reduces temperatures. For example, in underground nuclear disposal storage [3], temperature rises can be caused by chemical reactions or radiation, while in underground sequestration of carbon dioxide [4], injection of carbon dioxide could cool down the temperature of rock formation. In underground storage facilities for liquefied natural gas (LNG) [5], the temperature around the facilities could rise and fall over time with the flow of the LNG. Thermal energy can also be collected and stored when it is available and then used when needed. For example, hot water can be collected and stored in summer and then used in winter for heating purpose, or ice is collected and stored in winter and used in summer for cooling.

In some of these applications, the stability of the storage facilities is extremely important. Instability is mainly due to fracture initiation, propagation and coalescence. Thermal effect can make large contribution to the instability. For large-scale applications, physical experiments are very difficult, and usually impossible. Thus, numerical simulations have become vital tools for the design process. There are many numerical studies of thermal effects in various engineering applications. Just give a few examples. Numerical simulations have been undertaken for nuclear waste storage [5–8], underground sequestration of carbon dioxide [9,10], geothermal energy [11–15] and LNG underground storage facilities [16,17].

Among various numerical methods for deformation analysis, the displacement discontinuity method (DDM), an indirect boundary element method (IBEM), has an advantage over other numerical methods, especially for problems with cracks, in that a crack is discretised as one entity, reducing the number of final linear equations required to solve.

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Since Crouch [18] introduced DDM for study of cracks in two-dimension case, DDM has been studied and extended to different materials and to three dimensional cases. It has been used widely in numerical analysis of many engineering applications. We do not intend to give a detailed review of DDM, in steady, we just mention a few of the researchers in this field, for example, Crouch and Starfield [19] and Shen et al. [20] in two-dimensional cases and Wiles and Curran [21], Kuriyama and Mizuta [22], Shou et al. [23], Zhang et al. [24] and Shi et al. [25] in three-dimensional cases.

To utilise the advantage of less number of equations of DDM in simulations that take account of the thermal effect, particular for crack propagation, it would be better for the temperature field and the thermal effects on displacement and stress states to be solved using a similar indirect method. For two-dimensional applications, Shen et al. [20] developed the fracture propagation code (FRACOD) based on DDM, which can solve coupled thermo–hydro–mechanical problems and have been employed in simulations of many engineering applications. The fictitious heat source is treated as the basic variable for thermal effect. Ghassemi and co-workers [13–15,26,27] combined DDM and IBEMs for heat source and fluid pressure for three-dimensional poro-elastic medium. In most of the IBEMs for thermal field, fictitious heat strengths on boundary elements are taken as the basic variables, which are solved first from linear equations and then used to calculate temperature, displacement and stress at other points. Zhao et al. [28] used fictitious temperature discontinuity as the basic variable, instead of fictitious heat strength, combined with DD, to investigate the thermal effects. Most recently, IBEMs combined with some new techniques have been used for heat conduction, thermoelasticity and more general potential problems [29–31,47]. With BEMs, accurate computation of singular integrals and/or nearly singular integrals are crucial. The techniques used by Zhang et al. [32] are very useful.

We have been developing a DDM-based code FRACOD^{3D} [25] for three-dimensional problems. To incorporate the thermal effect in the code, we need expressions for displacements and stresses due to the heat source continuously acting on a planar triangular element. At the beginning, we were not able to find the final expressions, so we followed steps found in literature and derived an analytic expression for a three-dimensional thermal displacement potential function due to a constant heat source at a point in an infinite elastic body, which can be directly used to compute displacements and stresses. Then we were reminded that the expression was given in Nowacki's book [33]. After check, we found that the derivation in [33] is different from our procedure. So we think it may be worth to re-present the derivation here.

As in the DDM, it requires to integrate the expression of thermal displacement potential function due to point heat source over the (triangular) elements. A semi-analytic integration was first used for this purpose. The semi-analytical integration technique has been used by many researchers and it uses a polar coordinate system transformation for the two-dimensional integration, with integration with respect to radial variable expressed analytically. It eliminates the singularity that occurs when the integrand is evaluated on the triangle. However, we found that the integral values for vertices of the triangle and for points nearby computed with the semi-analytical scheme could be very different. This affects the accuracy of the derivatives of the potential function, and therefore accuracy of displacements and stresses, if they are evaluated by finite difference method at these points. This difficulty can be overcome by hybrid scheme of using direct numerical integration for these points and the semi-analytical scheme for other points away from the vertices. Rong et al. [34] reported the similar problems of the polar coordinate system transformation.

The paper is arranged as follows. In Section 2, we re-derive the expression for a thermal displacement potential function due to constant heat source acting at a point and on a planar region in an infinite, elastic, three-dimensional body. A hybrid scheme of a semi-analytical integration and a direct numerical integration on a triangle is shown in Section 3 to integrate the displacement potential function and temperature field. In Section 4, the numerical scheme of an IBEM for a temperature field and DDM for an elastic field with the thermal effect is outlined. Simulations of a verification example and a real underground LNG storage cavern are shown in Section 5. Our conclusions are given in Section 6.

2. Thermal displacements and stresses due to a heat source on a triangle

Temperature variation in elastic medium will change stresses and displacements within the medium. The temperature variation is due to heat source variation; therefore, variation of the heat source changes the stresses and displacements. In this section, we formulate the relationships between the heat source and the stresses and displacements. In particular, for application of the three-dimensional DDM, we formulate the relationships for a heat source on a triangular element.

2.1. Thermal displacements and stresses due to a constant point heat source

It is assumed that conduction is the only heat transfer mechanism for the temperature change. Thus, the temperature change T is governed by the heat conduction equation:

$$\chi\nabla^2 T = \chi\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)T = \frac{\partial T}{\partial t} \tag{1}$$

where $\chi = k/\rho c$ is the thermal diffusivity of the solid material [35], with k, ρ, c being thermal conductivity, mass density and specific heat capacity of the material, respectively; x, y, z and t represent the Cartesian coordinates of point and time, respectively. With thermal boundary conditions, the temperature field can be determined independently from the elastic deformation.

With the thermal elasticity constitutive relations [36,37]:

$$\sigma_{ij} = 2G\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij} - \beta T\delta_{ij},\ (i, j = x, y, z) \tag{2}$$

the equilibrium equations of the elastic medium are:

$$Gu_{i,jj} + (\lambda + G)u_{j,ji} = \beta T_{,i},\ (i, j = x, y, z) \tag{3}$$

in terms of displacement field u_i. In Eqs. (2) and (3), G is shear modulus and λ is a Lame's constant of the elastic body; $\varepsilon_{kk} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ is the volumetric strain; and $\beta = (3\lambda + 2G)\alpha/3$, with α being the volumetric thermal expansion coefficient of the elastic body under constant stress. Note that the volumetric thermal expansion coefficient is three times the value of the linear thermal expansion coefficient, used in some analyses; δ_{ij} is the Kronecker delta symbol; the subscript comma denotes derivatives with respect to coordinate; and the conventional summation over repeating index is used. In the constitutive relations (2), the strains are the classic ones, which are composed of partial derivatives of displacement components.

The general complete solution of (3) for the displacements has two parts: the complementary solution of the corresponding homogeneous equations, and a particular solution of the inhomogeneous equations (3) due to the temperature change T. Just from the governing equation's (3) point of view, the complementary solution can contain some arbitrary constants or functions, while the particular solution should satisfy the inhomogeneous equations (3) without any arbitrariness. The constants or functions in the complementary solution should satisfy the boundary conditions by the complete solution. In the following, we first consider the particular solution of equations in (3), i.e. the thermal displacements and stresses.

To determine the thermal displacement field due to the temperature change or the heat source, i.e. the particular solution of Eq. (3), a thermal displacement potential function, Φ, is introduced such that [38]:

$$u_i = \frac{\partial\Phi}{\partial x_i} \tag{4}$$

With the thermal displacement potential and temperature change, the thermal stresses are given by:

$$\sigma_{ij} = 2G\{\Phi_{,ij} - K_h T\delta_{ij}\} \tag{5}$$