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Transient numerical simulation of coupled heat and moisture flow through a green roof

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ABSTRACT

The paper reports a mathematical model governing unsteady coupled moisture and heat energy transport through a green roof, e.g. the canopy (leaf cover), the soil and the structural support. The mathematical model that governs the transport phenomena in the canopy is represented by a system of nonlinear ordinary differential equations (ODEs) for the unknown temperature of the plants, and the unknown temperature and moisture content of the canopy air. A set of nonlinear partial differential equations (PDEs) describe the heat and moisture transport through the soil and structural support. Continuous field functions such as temperature and relative humidity, are considered as the driving potentials. A finite difference numerical model is used to solve the ODEs and the boundary element numerical model is used to discretize the PDEs.

1. Introduction

Green roofs are specialized roofing systems that support vegetation growth on human-made structures such as rooftops [15]. The potential benefits of green roofs can be manifold and range from the functional, ecological, aesthetic, to storm water reduction and energy savings [5,12]. The potential for energy savings that green roofs can provide is important, and this is the motivation for the present work. The depth and the wetness of the growing medium/substrate significantly affect the heat transfer through the roof.

The objective of this paper is to present a numerical model applicable to simulating the dynamic heat and moisture transport behaviour of real green roofs that consist of three layers, these being the canopy (leaf cover), the soil and the structural support. The complexity of a canopy layer as a physical-biological system of heat and moisture transport is such that developing an exact mathematical model is almost impossible. In this paper we regard a canopy as a single homogeneous layer formed by a system of concentrated parameters, characterized by one value of leaf temperature and one value of temperature and moisture content in the canopy air [5,12]. Therefore, the canopy is represented by a system of nonlinear ordinary differential equations (ODEs) for the unknown temperature of the vegetation, and the unknown temperature and moisture content of the canopy air.

The growing substrate/soil and structural support were treated as an unsaturated homogeneous porous medium, where the heat and moisture transport is interlinked and coupled, which is quantitatively governed by a set of coupled nonlinear partial differential equations (PDEs). Coupled heat and moisture transfer in a rigid porous medium under temperature and moisture gradients has been extensively studied [5,13,21].

The finite difference model [9] is used to solve the ODEs and the boundary element model [10] is used to discretize the PDEs. Numerical solutions of the green roof model are strongly related to the plant and soil parameters defining thermal energy transport, mass transport and storage behaviour, empirical relations involved in modelling heat and moisture fluxes, crop transpiration, choosing relevant different stem or root lengths of the plants and characteristics of the canopy, etc. In general, the empiricism applied in this contribution was taken from [3–5,12,18,20]. The parameter values vary considerably and have a great impact on the numerical solutions.

2. Governing conservation equations for the canopy

The canopy (c) is composed of the leaves/plants (p) and the canopy air (ca) within the leaf cover. The complexity of a canopy transport phenomenon as a thermodynamic system of heat and mass sources and sinks is such that an exact mathematical physical description is almost

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impossible. In this paper we have taken a canopy to be one homogeneous layer, characterized by one temperature value for leaves (T_p) and one temperature and vapour pressure value of canopy air, T_{ca} and $p_{v,\,ca}$, respectively. Such a layer is bounded by the soil ground (g) at the bottom, and an ideal air surface (a) at the top. Under these assumptions, the set of macroscopic ODEs governing the energy and mass transport phenomena in the canopy plants and canopy air can be formulated [2,5,12,15].

2.1. Heat energy conservation equation for the plant

The canopy plant heat energy balance equation describing accumulation within a control volume, the shortwave radiation (*rs*), longwave radiation (*rl*), convection and transpiration heat fluxes, (e.g. the short wave solar radiation received by plant leaves q_p^{rs}, long wave heat exchange q_p^{rl} with the sky and with the soil ground), convective heat exchange with canopy air q_{ca-p}^{conv}, convective heat exchange with ambient air q_{a-p}^{conv}, and heat loss due to transpiration heat flux q_{p-ca}^{trans}, can be formulated as [4,5,15,20]

$$c_p \delta_l LAI \frac{dT_p}{dt} = q_p^{rs} + q_p^{rl} + q_{ca-p}^{conv} + q_{a-p}^{conv} - q_{p-ca}^{trans}, \tag{1}$$

where the primitive variable in Eq. (1) is the temperature of the plant $T_p(t)$, whilst the quantities $c_p = (\rho c_p)_p$, δ_l, and *LAI* represent the effective specific heat per unit volume, the average leaf thickness, and the leaf area index, which is the ratio of the total of leaf top surface area to the ground below, i.e. (leaf area)/ (substrate surface), respectively.

Using constitutive models to express heat fluxes in Eq. (1), the short wave solar radiation absorbed by the plant is given by

$$q_p^{rs} = \sigma_f [(1-\tau_s - \rho_p^r)(1+\tau_s \rho_g^r) q_{sol}] \quad \text{and} \quad \rho_p^r = (1-\tau_s)\rho_\infty^r, \tag{2}$$

where q_{sol} represents the solar radiation at the top of the canopy, τ_s is the shortwave transmittance of a canopy, ρ_p^r, ρ_∞^r and ρ_g^r are the shortwave reflectances of leaves, a dense canopy and a soil surface/ground, respectively, and σ_f is a fractional vegetation coverage. Shortwave transmittance is based on *LAI* and coefficient of transmittance/extinction for shortwave radiation k_s is written as

$$\tau_s = \exp(-k_s LAI). \tag{3}$$

The longwave radiation heat flux q_p^{rl} is the sum of longwave heat exchange with sky q_{sky-p}^{rl} and longwave heat exchange with the soil ground q_{g-p}^{rl} and can be expressed as

$$q_p^{rl} = q_{sky-p}^{rl} + q_{g-p}^{rl} = \sigma_f [\alpha_{sky-p}^{rl}(T_{sky} - T_p) + \alpha_{g-p}^{rl}(T_g - T_p)], \tag{4}$$

where T_{sky} is the sky temperature and T_g is the soil surface temperature. The linearized longwave radiation heat transfer coefficients are given as

$$\alpha_{sky-p}^{rl} = 4\varepsilon_p (1-\tau_l)\sigma \left(\frac{T_{sky}+T_p}{2}\right)^3,$$
$$\alpha_{g-p}^{rl} = 4\varepsilon_{g-p} (1-\tau_l)\sigma \left(\frac{T_g+T_p}{2}\right)^3, \tag{5}$$

where τ_l is the longwave transmittance/extinction of a canopy

$$\tau_l = \exp(-k_l LAI) \tag{6}$$

and k_l is the longwave radiation transmittance/extinction coefficient. The effective emissivity ε_{p-g} between the plant and the soil is expressed as

$$\varepsilon_{g-p} = \frac{1}{\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_g} - 1}, \tag{7}$$

where the quantities ε_p and ε_g are the leaves and soil emissivity, respectively.

Convection heat fluxes such as the exchange between canopy air and plant and that between ambient air and plant are given by the following expressions:

$$q_{ca-p}^{conv} = 2\alpha_{ca-p}(T_{ca} - T_p) \quad \text{and} \quad q_{a-p}^{conv} = \sigma_f \alpha_{a-p}(T_a - T_p). \tag{8}$$

Transpiration heat loss flux is the one-way heat exchange from the plant to the canopy air due to vapour pressure deficit $p_{v,\,p} > p_{v,\,ca}$ and is given by

$$q_{p-ca}^{trans} = 2\alpha_{p-ca}^{trans}(p_{v,p} - p_{v,ca}), \tag{9}$$

where $p_{v,\,p}$ is the vapour pressure at the leaf tissues equal to saturated vapour pressure $p_{v,p} = p_v(T_p)$. The constant (2) in Eqs. (8) and (9) accounts for the lower and upper leaf surfaces.

The heat transfer coefficients α_{ca-p}, α_{a-p} and α_{p-ca}^{trans} can be formulated as follows:

$$\alpha_{ca-p} = \frac{c_{ca}}{r_e} LAI, \quad \alpha_{a-p} = \frac{c_a}{r_{ea}} \quad \text{and} \quad \alpha_{p-ca}^{trans} = \frac{c_{ca}}{\gamma(r_e + r_i)} LAI, \tag{10}$$

where the quantities $c_{ca} = (\rho c_p)_{ca}$ and $c_a = (\rho c_p)_a$ are the specific heat per unit volume of the canopy air and ambient air, respectively, the quantities r_e and r_i express the external and internal canopy resistance to heat transfer, r_{ea} is the aerodynamic resistance to heat transfer with the surrounding environment, whilst the quantities σ_f and γ are the fractional vegetation coverage and the thermodynamic psychometric constant.

2.2. Heat energy conservation equation for the canopy air

The canopy air heat energy balance equation describing accumulation within the control volume, the convection heat fluxes, i.e. the convective heat exchange with the ground q_{g-ca}^{conv}, the convective heat exchange with the ambient air q_{ca-a}^{conv} and the convective heat exchange with the plant $q_{p-ca}^{conv} = -q_{ca-p}^{conv}$ is [4,5,12,20]

$$c_{ca} L_c \frac{dT_{ca}}{dt} = q_{g-ca}^{conv} + q_{a-ca}^{conv} + q_{p-ca}^{conv}, \tag{11}$$

where the primitive variable in Eq. (11) is the temperature of the canopy air $T_{ca}(t)$ and L_c represents the average canopy/plant height, respectively.

Using constitutive models to express heat fluxes in Eq. (11) we can write the convective heat flux with the ground and ambient air, given by

$$q_{g-ca}^{conv} = \alpha_{g-ca}(T_g - T_{ca}) \quad \text{and} \quad q_{a-ca}^{conv} = \alpha_{a-ca}(T_a - T_{ca}). \tag{12}$$

The heat transfer coefficients α_{g-ca} and α_{a-ca} are given by the following empirical formulas:

$$\alpha_{g-ca} = \frac{c_{ca}}{r_{g-ca} + r_s} \quad \text{and} \quad \alpha_{a-ca} = \frac{c_a}{r_{ea}}, \tag{13}$$

where the quantities r_{g-ca} and r_s express the soil surface to canopy air resistance and additional moisture dependent soil surface resistance, respectively.

2.3. Moisture mass conservation equation for the canopy air

The moisture mass balance equation of the canopy air describing accumulation of the vapour within control volume and the vapour mass fluxes account for transpiration from the plants $j_{v,p-ca}^{trans}$, evaporation from the ground $j_{v,g-ca}$ and vapour exchange with the ambient air $j_{v,a-ca}$, and it can be written as [4,5,12,20]

$$\rho_{ca} L_c \frac{d\omega_{ca}}{dt} = j_{v,p-ca}^{trans} + j_{v,g-ca} + j_{v,a-ca}, \tag{14}$$

where the primitive variable in Eq. (14) is the absolute or specific humidity of the canopy air $\omega_{ca}(t)$ and the quantity ρ_{ca} is the canopy air mass density. Since the specific humidity can also be expressed as