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On elastoplastic analysis of some plane stress problems with meshless methods and successive approximations method

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A B S T R A C T

A numerical method based on the method of fundamental solutions (MFS) and the method of particular solutions (MPS) together with the successive-approximation iteration process is presented. The nonlinear behaviour of the material that hardens with plastic deformation is characterized by the Chakrabarty model. The considerations are based on the incremental theory of plasticity. Furthermore, the incremental strain equations relate the plastic strain increments to the total strains only (the stresses do not appear there). The method is used for solving three example boundary value problems that describe the stress state in some plates subjected to external loads. The accuracy of the results is examined on the basis of the boundary conditions fulfilment and the comparison with the finite element method (FEM). Finally, the regions of elastic/plastic deformation are identified. Then, the distribution of the equivalent stress is shown.

1. Introduction

Elastic–plastic problems are of special interest in the engineering applications. For the design of large structures or even structural elements under some load conditions, a knowledge of stresses acting within these elements and also properties of a material are required. A common approach assumes a linear relationship between stresses and strains and its mathematical bases are provided by the theory of elasticity (see, e.g., [1,2]). An important extension of this approach is the theory of plasticity [3–8] that enables an analysis of stresses and strains that appear in structural elements in both elastic and plastic ranges, respectively. The response of a material beyond the load corresponding to the yield stress can be modelled by linear or nonlinear stress–strain relations. The more comprehensive constitutive model we use, the better design of a structure considered.

The considered mechanical problems are formulated in the form of boundary value problems (BVPs) for partial differential equations (PDEs) with the governing equations that are nonlinear ones. There are many numerical methods that can be used to solve the elastic–plastic problems. The most commonly used ones are the finite element method (FEM) [9–13], the boundary element method (BEM) [14–17] and the finite difference method (FDM) [18]. They all belong to a group of so called mesh methods. The approximate solutions of these problems can be also obtained with meshless methods that gained popularity because of their significant advantages. Among the most important ones we can distinguish the following issues. A cloud of points only (i.e., a set of their coordinates) is required for the meshless methods instead of a mesh (grid) of points that has to be built in the case of mesh methods. An implementation is very simple even for problems defined in complicated geometries and/or in three dimensions. An approximation of the solution is proposed as a linear combination of radial basis functions (RBFs). This approach is convenient because of the fact that the approximate solution is given as a continuous function with continuous derivatives. We can make a practical and effective use of this property when a given physical quantity is represented by a derivative of the solution function.

Among the most recent works in the area of elastic–plastic problems we can mention the following ones. Tsiatas and Babouskos [19] analyzed the elastic–plastic problem of functionally graded bars subjected to torsional loading. A formulation of the problem is general and can be applied to bars of an arbitrary cross-section. The authors used the BEM and the analog equation method [20] for computations. Assidi et al. [21] also considered some structural plasticity problems. They used the algorithm belonging to a family of asymptotic numerical methods (ANM) [22] to solve a number of elastic–plastic problems (e.g., a stretching of a rectangular plate, a bending of a cantilever beam, an uniaxial tension of a plate with a central circular hole). Similarly, in [23], Zahrouni et al. chose the deformation theory of plasticity and applied the ANM to solve selected problems involving nonlinear constitutive laws. Then, in [24], Foti and di Roseto modelled analytically and with the FEM the elastic–plastic behaviour of metallic strands under axial-torsional loads. Hassani and Faal [25] took into account the Saint-Venant torsion of orthotropic bars with rectangular cross section weakened by cracks. The authors solved several examples of an arc-crack and a single straight crack. They also studied the interaction between

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two cracks. In [26], Sapountzakis and Tsipiras used the BEM to solve a nonlinear inelastic uniform torsion of cylindrical composite bars of arbitrary cross section consisting of materials in contact. In [27], Uomoto et al. applied the mesh-independent data point finite element method (MDP-FEM) for large deformation elastic–plastic problems. As an example, they analyzed the diffused necking of tensile bars. Finally, in [28], Yoon et al. presented an efficient warping model for elastoplastic torsional analysis of composite beams.

There are a number of meshless methods that can be chosen for solving elastic and elastoplastic problems. In the case of elastoplastic ones, the appropriate algorithms are dedicated for the nonlinear problems. We can mention, e.g., the MFS–MPS that is often used with the Picard iteration process [29,30], the Kansa-RBF method for the nonlinear problems [31,32], the homotopy analysis method (HAM) [33–35] or one of some other iteration processes [18,36]. In [37], Liu and Gu presented a point interpolation method (PIM) that was further used for two-dimensional solids. Wen and Aliabadi [38] presented an improved meshless collocation method that can be used for higher order derivatives of shape functions by radial basis function method or moving-least square method. The authors applied this approach for elastostatic and elastodynamic problems. Then, in [39], Tu et al. analyzed material nonlinearity with an effective shear modulus approach. This method is based on the strain control method by the use of point collocation method. The authors applied it for solving two-dimensional elastoplastic problems. Next, in [40], Kolodziej and Fraska took into account an elastic torsion of bars possessing a regular polygonal cross-section by means of boundary collocation method. The authors analyzed a few cases of bars, e.g., simply connected rods, two connected rods (rods with a hole), and composite bars (two different materials) possessing a regular polygon in their cross-section contour. In [41,42], Karageorghis et al. used the MFS for the solution of some inverse void (rigid inclusion or cavity) problems that arise in two-dimensional isotropic linear elasticity. In [43], Kolodziej and Gorzelanczyk also studied a torsion of prismatic bars. They assumed an elastic–plastic material behaviour, the Saint-Venant displacement assumption and deformation theory of plasticity. In [29] Kolodziej et al., applied the MFS–MPS for the inverse problem related to the determination of elastoplastic properties from the torsional experiment. Then, Jankowska and Kolodziej [30,36] applied the MFS–MPS for the study of the stress state in a plate subjected to elastic–plastic deformation due to uniaxial extension. In [30], the problem was formulated with the deformation theory of plasticity and the Ramberg–Osgood stress–strain equation. In [36], the authors applied the incremental theory of plasticity with the associated flow rule given by the Prandtl–Reuss relations and the stress–strain model proposed by Chakrabarty [4]. In [44], Karageorghis et al. presented the matrix decomposition algorithms for the efficient solution of the linear systems arising from Kansa-RBF discretizations of elliptic boundary value problems in regular polygonal domains. The authors considered the Poisson equation, the inhomogeneous biharmonic equation, and the inhomogeneous Cauchy–Navier equations of elasticity. Then, in [31,32], Jankowska et al. applied the Kansa-RBF collocation method to two-dimensional nonlinear boundary value problems. Among the others, an elastoplastic torsion problem (see also [29]) was taken into account. The authors solved the nonlinear system of equations resulting from the Kansa-RBF discretization for the unknown coefficients in the RBF approximation by a method of nonlinear optimization.

Subsequently, we consider the numerical method based on the MFS–MPS and the successive-approximation iteration process for solving some elastic–plastic plane stress problems. Since it is well-known that the plastic strains are dependent on the loading path, the incremental theory of plasticity is taken into account. The research is based on the approach proposed in [18], where incremental strain relations relate the plastic strain increments to the total strains only and the stresses do not appear there. For the nonlinear stress–strain relationship, we choose the model proposed by Chakrabarty [4]. As it is a nonlinear relationship in the range of plastic deformation with a parameter that is a strain-hardening exponent, we can use it to model a behaviour of many materials. As numerical examples we take the boundary value problems describing the stress state in plates of geometries such that the regions of stress concentration can be observed. The plates are subjected to uniaxial extension or compression. There is a number of publications (see e.g. [8,18,45,46]) that consider the boundary value problem with a governing equation of the form presented in this paper. The authors use the iteration process defined in [18], but the appropriate successive approximations are obtained with some finite difference method. In our paper, we propose an algorithm based on meshless methods, i.e. the MFS–MPS, where the partial derivatives of the plastic strain increments are computed with the generalized finite differences. Such an approach was not previously proposed for this boundary-value problem. In addition to the advantages which are common for a class of meshless methods, we can indicate the ones that are specific for the algorithm considered. For example, we can significantly reduce the dimension of the linear system of equations solved in each step of the iteration process. In the case of our method the dimension of the matrix of coefficients required by the MPS depends only on the number of interpolation points and polynomials. We also have to solve another system of equations due to the MFS applied, but its dimension depends only on the number of boundary and source. For the finite difference method all mesh points are taken into account when the matrix of coefficients is built.

The paper has the following layout. The stress–strain relations, the constitutive model and the problem formulation are described in Section 2. Then, in Section 3 we propose the algorithm based on the successive-approximation iteration process [18] and the meshless methods, i.e., the MFS–MPS. Since in the right-hand side function of the governing equation, the partial derivatives of the plastic strain increments are included, we also use the generalized finite differences [47–54] for computations. A sequence of numerical results concerning the solution of the example problems is presented in Section 4. The exactness of the results obtained with the considered algorithm is examined on the basis of the boundary conditions fulfilment and the comparison with the FEM. Then, the distribution of the equivalent stress and the regions of elastic and plastic deformation are shown. The conclusions listed in Section 5, bring the paper to the end.

2. Problem formulation

2.1. Assumptions about stress–strain relations and elastic–plastic constitutive model

We focus on incremental strain relations given in the form proposed by Mendelson [18] (see also [8,45,46]). These equations relate the plastic strain increments to the total strains only and they do not recourse to the stresses.

We assume that a loading path to a specific state of stress is divided into K increments of load. For a given kth increase of the load by a small amount, a plastic strain increment $\Delta\varepsilon_{ij}^{p}$ is produced. Hence, the total strain ε_{ij} can be written as

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} + \Delta\varepsilon_{ij}^{p}, \quad \varepsilon_{ij}^{p} = \sum_{m=1}^{k-1} \Delta\varepsilon_{ij,m}^{p}, \tag{1}$$

where ε_{ij}^{e} is the elastic component of the total strain, ε_{ij}^{p} is the accumulated plastic strain up to (but not including) the current increment of load and $\Delta\varepsilon_{ij}^{p}$ is the increment of plastic strain due to the current increment of load. Following the derivation presented in [18], we define the modified total strains as

$$\varepsilon_{ij}' \equiv \varepsilon_{ij} - \varepsilon_{ij}^{p}. \tag{2}$$

Substituting Eqs. $(1)_1$ to (2), we obtain

$$\varepsilon_{ij}' = \varepsilon_{ij}^{e} + \Delta\varepsilon_{ij}^{p}. \tag{3}$$

Subtracting the spherical strain tensor $\varepsilon_m\delta_{ij}$ (in which $\varepsilon_m = \frac{1}{3}\varepsilon_{ss}$ denotes the mean strain) from both sides of Eq. (3), we get the modified strain