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Reliability analysis of seepage using an applicable procedure based on stochastic scaled boundary finite element method

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ABSTRACT

This paper presents a practical approach for reliability analysis of steady-state seepage by modeling spatial variability of the soil permeability. The traditional semi-analytical method; named Scaled Boundary Finite-Element Method (SBFEM) is extended by a coded program to develop a stochastic SBFEM coupled with random field theory. The domain is discretized into several non-uniform SBFEM sub-domains. The flow quantities such as exit gradient, flow rate, and the reliability index of piping safety factor are estimated. The precision of the outputs and the accuracy of the method are verified with the Finite-Element Method (FEM). A set of stochastic analysis is performed in three illustrative examples to illuminate the applicability of the proposed method. In these examples, the effect of the variations in the position of the sub-domain discretization center, the cutoff location, and the cutoff length are investigated stochastically. Further, the influence of the permeability's Coefficient of Variation (COV_k) and the correlation length is evaluated. The results are shown acceptable agreement with those obtained by the conventional Stochastic Finite-Element Method (SFEM). The proposed approach has potential to model the complex geometries and cutoffs in different locations without additional efforts to deal with the spatial variability of the permeability.

1. Introduction

Seepage analysis has a crucial importance in geotechnical engineering. The flow results play a key role in assessment of flow rate, exit gradient, and safety factor against piping. The mentioned quantities may be required for the safe design of various engineering structures such as dams, bridges and retaining walls. Over the last decades, several methods such as classical, analytical, and numerical methods have been utilized to analyze seepage problems. A succinct explanation of each method with relevant flaws and recent contributions is presented in the following.

The electrical analogue is of classical methods. Ohm's law of an electric current is the counterpart of Darcy's law in a flow regime. Limited size of electro-conducting materials seriously restricts the method performance in large-scale problems [1]. The flow net is a drawing classical solution. In fact, the flow net is a plot of two perpendicular sets of streamlines and equipotential lines which can be obtained by tedious sketching techniques through trial-and-error [1].

Although analytical methods are required to attain a profound concept of the basic physics [2], a few cases with simple geometry can be solved analytically. He [3] has proposed a model for seepage flow in porous media with fractional derivatives.

Numerical approaches are capable of analyzing more complicated seepage problems; however, these methods still have some pertinent deficiencies. The Finite-Difference Method (FDM) [4,5], the Finite-Volume Method (FVM) [7,8], the Finite-Element Method (FEM) [9–13], the Boundary Element Method (BEM) [14–16] and meshless methods [17–19] are used as the most conventional numerical procedures in seepage problems.

The FDM is the earliest numerical procedure to solve partial differential equations. The studies of Bardet and Tobita [4] and Jie et al. [5] can be named among the important contributions using the FDM. Bardet and Tobita [4] presented a practical FDM for unconfined seepage. Jie et al. [5] applied FDM to deal with steady seepage analysis in the homogeneous isotropic medium. The FVM is mainly employed for the numerical solution of problems in fluid mechanics that was introduced by McDonald in 1971 [6]. Darbandi et al. [7] developed a moving-mesh FVM capable of solving the seepage problem in domains with arbitrary geometries. Bresciani et al. [8] applied a method based on a finite volume scheme to analyze steady-state flow in porous media. Like the FD and FV methods, the FEM is a mesh generation based approach, which has substantial restrictions to model singularities related to adjacency of sharp edges and angles. Bathe and Khoshgoftaar [9] used the FEM to study the seepage through porous media. Ataie-Ashtiani et al. [10] employed the FEM to simulate the groundwater flow in unconfined aquifers

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with a periodic boundary condition. Ouria and et al. [11] studied a nonlinear analysis of transient seepage beneath a dam considering the effect of the change in permeability of soil by the coupled FEM. Chen et al. [12] proposed a numerical solution based on the FEM to solve complex drainage systems. Kazemzadeh-Parsi and Daneshmand [13] numerically analyzed three-dimensional unconfined seepage problems in homogeneous and anisotropic domains with arbitrary geometry using the fixed grid finite-element method.

As a feature of the BEM, complicated geometries are simply discretized just by boundary discretization. However, a fundamental solution is needed to satisfy the governing equations in the domain. This is regarded as the main defect of this approach. The investigations of Brebbia and Chang [14], Chen et al. [15], and Rafiezadeh and Ataie-Ashtiani [16] can be named as the outstanding contributions. Brebbia and Chang [14] applied boundary elements to seepage problems in inhomogeneous and anisotropic soil mediums. Chen et al. [15] employed the BEM based on the formulation of the dual integral equations to analyze the seepage flow under a dam with sheet piles. Rafiezadeh and Ataie-Ashtiani [16] used the BEM to analyze transient free-surface seepage problems in anisotropic materials.

Meshless methods are used to solve fluid mechanic problems by utilizing unstructured nodes. The accuracy of these approaches is acceptable, howbeit they are well known as time-consuming methods that is a remarkable imperfection. Hashemi and Hatam [17] utilized radial basis function-based differential quadrature method as a mesh-free approach to solve two-dimensional transient seepage. Jie et al. [18] investigated the application of the natural element method in seepage analysis with a free surface. Zhang et al. [19] combined the moving Kriging interpolation and the Monte Carlo integration to analyze unconfined transient seepage through homogeneous and inhomogeneous media.

Recently, Song and Wolf [20] have developed the SBFEM to overcome the limitations of the previous methods. It is a novel semi-analytical approach for the dynamic analyzing of unbounded domains. This method combines significant advantages of the FEM and the BEM. Over the last two decades, the SBFEM has been successfully applied to various problems in bounded and unbounded domains [21-24]. Nevertheless, just a few studies focused on seepage problems using the SBFEM. Bazyar and Graili [25] extended SBFEM to analyze confined and unconfined seepage problems. Bazyar and Talebi [26,27] applied the SBFEM to analyze transient seepage and heat conductivity in anisotropic soils.

In the mentioned studies, the sources of uncertainty of input parameters were not considered and seepage problems were analyzed deterministically. However, due to the inherent uncertainties of the soil characteristics, indicate that seepage problem has a probabilistic nature rather than deterministic. Thus, a stochastic assessment of seepage analysis would be useful as an alternative or a supplement of the deterministic analysis to provide better engineering decisions. Griffiths and Fenton [28–30] studied the effect of stochastic soil permeability on confined seepage beneath water retaining structures based on spatially random soil for two-dimensional and three-dimensional seepage problems. Ahmed [31] extended stochastic analysis of free surface flow through earth dams by the FEM. Ahmed et al. [32] investigated the problem of confined flow under dams and water-retaining structures using stochastic finite-element modeling in anisotropic heterogeneous soils. Srivastava et al. [33] studied the influence of spatial variability of the permeability property on steady state seepage flow and slope stability analysis using finite difference numerical code. Luo et al. [34] expressed a simplified procedure for reliability analysis considering the spatially varying of soil parameters. Rohaninejad and Zarghami [35] combined the Monte Carlo and the finite difference methods to evaluate the physical behavior of embankment dams stochastically. Cho [36] probabilistically analyzed the seepage to consider the spatial variability of permeability for an embankment on soil foundation in a layered soil profile. Long et al. [37] explored a sensitivity analysis of the SBFEM for the elastostatics. Long et al. [38] analyzed the fracture of cracked structures stochastically considering the random field properties by the SBFEM. Ahmed et al. [39] scrutinized the flow beneath water-retaining structures under heterogeneous conditions. Hekmatzadeh et al. [40] investigated the effect of uncertainty in the soil properties, earthquake coefficients, and sediment characteristics in the stability of a diversion dam.

The common feature of the mentioned studies was the consideration of spatial variability for simple problems, while they had their corresponding constraints especially for complex geometry, sharp corners, and singular points. The main goal of this paper is to overcome the mentioned limitations and develop a method for reliability analysis of the steady-state seepage using stochastic SBFEM to consider the spatial variability of the permeability. On one hand, it is worth mentioning that the typical SBFEM is not capable of discretizing the whole domain except the boundary. On the other hand, the entire domain needs to be discretized to take into account the spatially varying of soil properties due to random field theory. For this purpose, a coded computer program is required to couple the SBFEM with random field theory, which is provided in this research. The outputs of the proposed method are verified by FEM solution in another coded program. In further part of this study, to clarify the efficiency and accuracy of the proposed method in reliability analysis of seepage flow, three stochastic examples are conducted. The influence of variations in the location of sub-domain discretization center, the cutoff location, and the cutoff length are elucidated in the examples. Furthermore, the effect of the COV_k and the correlation length on the groundwater flow quantities are investigated as well.

2. Steady-state two-dimensional seepage

The flow of water within the pores of materials is an intricate phenomenon. Henry Darcy discovered an efficient law to properly explain that how water flows through the porous medias. Darcy's law attributes the velocity and the rate of water flow in a porous medium to the hydraulic gradient and the permeability coefficient as [41]:

$$v_X = -k_X \frac{\partial h}{\partial X} \tag{1}$$

where v_X stands for velocity in the X direction, k_X represents the permeability in the corresponding direction, and h holds the values of the hydraulic heads. The seepage problems can be solved based on boundary conditions. In one-dimensional problems, the water flows through direct columns of soil. In this case, Darcy's law can be applied to analyze the seepage problems directly. However, in two-dimensional conditions like the seepage beneath dams and water retaining structures, nonlinear streamlines are constructed. Thus, Darcy's law cannot be exerted straightly. Therefore, driving a governing differential equation is essential to solve the seepage problems. The detailed elucidation of pertinent formulations is expressed in Mariño's book [41]. The two-dimensional fundamental differential equation of seepage flow in porous media can be expressed as [41]:

$$k_h \frac{\partial^2 h}{\partial X^2} + k_v \frac{\partial^2 h}{\partial Y^2} = S_s \frac{\partial h}{\partial t} \tag{2}$$

where X and Y indicate the horizontal and vertical directions. k_h and k_v are the permeability in the horizontal and vertical directions, respectively. S_s is specific storage coefficient, and t stands for the time. For steady-state groundwater flow, the specific storage coefficient and time conditions are vanished due to the conservation of water mass. Therefore, Eq. (2) transforms into the Laplace's equation as [42,40]:

$$\frac{\partial}{\partial X}\left(k_h \frac{\partial h}{\partial X}\right) + \frac{\partial}{\partial Y}\left(k_v \frac{\partial h}{\partial Y}\right) = 0 \tag{3}$$

A solution of Laplace's equation is required to analyze the steady-state groundwater flow problems. The output of Eq. (3) is the values of hydraulic head. Some important seepage quantities such as flow rate, exit gradient, and safety factor against piping can be assessed using determined values of hydraulic head.