Engineering Analysis with Boundary Elements 94 (2018) 69–78

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

Evaluation of the T-stress and stress intensity factor for multi-crack problem using spline fictitious boundary element alternating method

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ARTICLE INFO

Keywords:
Spline fictitious boundary element alternating method
T-stress
Stress intensity factor
Muskhelishvili's fundamental solutions
Multi-crack problem

ABSTRACT

In this paper, the T-stress and stress intensity factor (SIF) of multiple cracks with arbitrary position in a finite plate is evaluated by the spline fictitious boundary element alternating method. The multi-crack problem is firstly divided into a simple model without crack which can be solved by the spline fictitious boundary element method and several infinite domains with one crack which can be solved by the fundamental solution of an infinite domain with a crack, namely Muskhelishvili's fundamental solutions. The technique is superior as no meshing is needed near crack face and the analytical solution for solving infinite domains with one crack is accurate and efficient. Then, instead of using the asymptotic expansion, the closed-form expression for calculating the T-stress in multi-crack problem is derived directly, which makes it convenient and accurate for calculating the T-stress. Besides, the SIF can be calculated using the analytical SIF expression in Muskhelishvili's fundamental solutions. Finally, T-stresses and SIFs in a numerical example with double cracks are computed to validate the accuracy of the presented method, And the other two examples with three cracks are further studied to investigate the influence of lengths and locations of multiple cracks on their T-stresses and SIFs.

1. Introduction

In the classical fracture mechanics, the stress intensity factor (SIF) is an important parameter to characterize the stress and displacement fields in the vicinity of crack tips as the parameter is considered to be sufficient for linear elastic fracture analysis [1]. However, numerous studies [1–5] show that the first non-singular term in the William's expansion [6], regarded as the T-stress, also plays an important role in the linear elastic fracture analysis of brittle materials. The major application of the calculation of T-stress is the prediction of crack growth path and stability [1,2,4,7,8]. What's more, the work in [9–11] shows that the sign and magnitude of the T-stress can seriously affect the size and shape of the plastic zone. Therefore, many researchers believe that the behavior of the crack tips is controlled by the combination of the SIF and the T-stress [12–15].

The T-stress is widely calculated by the finite element method (FEM), including crack problems under mode I and mixed mode I/II loading [16], test specimens subjected to non-uniform stress distributions [17] and cracks in functionally graded materials [18]. The boundary element method (BEM) is also applied to the determination of T-stress as it has a great advantage in the field of elastic fracture mechanics analysis. Phan [1] introduced a non-singular boundary integral formula to determine the T-stress for cracks of arbitrary geometry. Quarter-point crack-tip elements was used by Tan to solve T-stress in BEM [19], then the method was applied to the analysis of the multiple edge cracks in thick-walled cylinders [20]. Shah [21] evaluated the T-Stress for an interface crack lying between dissimilar anisotropic solids using BEM. Cheng [22] analyzed the role of the non-singular stress in brittle fracture by BEM coupled with eigen-analysis. In addition to the two methods above, numerous approaches such as boundary collocation method [23], technique of using dislocation arrays [24], scaled boundary finite-element method [25] and finite element based interior collocation method [26] have also been employed to calculate the T-stress.

However, as stated in [27], the review of the literature indicates that most methods for obtaining T-stress are confined to single crack problems with simple geometry and loading conditions, which limits the application of T-stress in engineering.

The alternating method for solving multiple crack problems is proposed in recent years. The domain with multiple cracks is considered as an overlapping region of two kinds of regions: namely the domain without crack and several domains with a single crack, then these two different problems can be solved separately, which can avoid the crack tip processing, greatly improving the solving efficiency. The alternating method is recently extended to calculate the T-stress of multi-crack problems. The T-stress of an infinite plate with two cracks was obtained by decomposing the original problem into a uniform stress field and many

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https://doi.org/10.1016/j.enganabound.2018.06.004
Received 20 February 2018; Received in revised form 4 May 2018; Accepted 9 June 2018

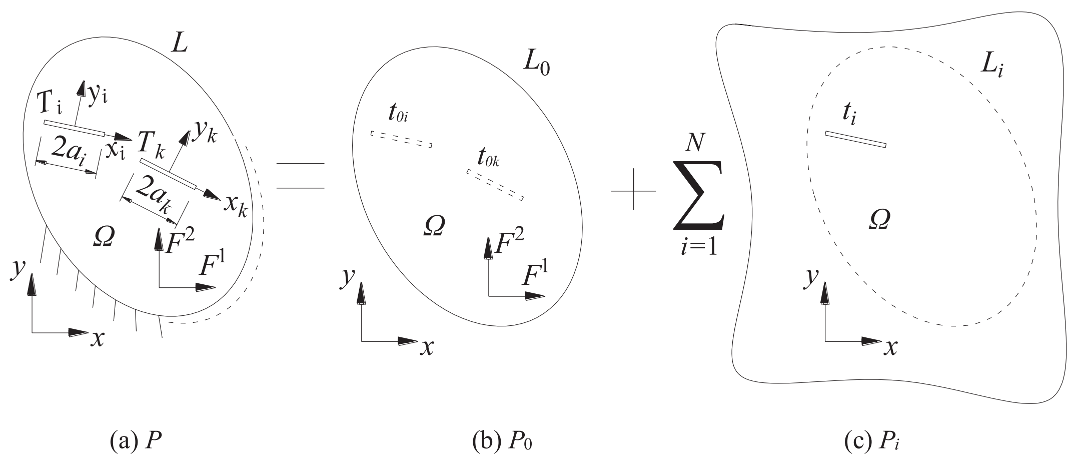

Fig. 1. Alternating method for multiple crack problem.

single crack problems [28]. The interaction between the T-stress results of two cracks with arbitrary position in an infinite plate was explored [29]. An alternating method based on two kinds of integral equations is proposed and the method is used to obtain the T-stress and SIF of a finite plate with two internal cracks [30,31]. The singular integral equation was introduced in alternating method for an infinite plate with multiple cracks [27]. Singular integral equation and Green's function method was used to evaluate the T-stress of an elastic half-plane with two internal cracks [32].

In the literatures mentioned above, two technologies are used to solve the T-stress. Firstly, an extrapolation formula is used to obtain the value of T-stress, which brings new numerical errors in the calculation process, and the precision of solutions is very sensitive. Secondly, a non-singular boundary integral equation (BIE) [1] is derived for the T-stress using the asymptotic stress expansion in the vicinity of a crack tip, it can numerically evaluate the T-stress without using any extrapolation formula. However, both two technologies are numerical methods, and the numerical error is unavoidable. What's more, all the literatures using alternating method have not mention the multiple crack problems with edge crack.

In this paper, in order to obtain the T-stress of multiple cracks with arbitrary position in a finite plate, the spline fictitious boundary element alternating method (SFBEAM) is proposed. Firstly, the finite plate with multiple cracks is divided into a finite domain without crack and several infinite domains with one crack. The finite domain without crack can be solved by the spline fictitious boundary element alternating method (SFBEM), only boundaries need to be partitioned in SFBEM thus the amount of calculation is very small. The infinite domains with one crack can be solved by analytical solutions, named Muskhelishvili's fundamental solutions, which guarantees the accuracy and efficiency. Then, in order to calculate the T-stress, the closed-form expression is derived directly, which makes it convenient and accurate for evaluating the T-stress in multiple crack problems. Besides, by using the analytical SIF expression in Muskhelishvili's fundamental solutions, the SIF can be calculated. Finally, a numerical example with double cracks for computing T-stresses and SIFs are presented to verify the accuracy of the presented method comparing with the solutions from literature and singular finite element method (SFEM). And, the other two examples with three cracks are further studied to investigate the influence of lengths and locations of multiple cracks on their T-stresses and SIFs.

2. Computational model for the multi-crack problem using an alternating method

A finite domain Ω with N cracks is shown as Fig. 1(a), donated as problem P. The length of the kth crack is $2a_k$ ($k=1, 2, \ldots, N$), the loads on the kth crack face is denoted by $T_k = p(x_k)\text{-}iq(x_k)$, where $q(x_k)$ are the normal stresses and $p(x_k)$ are the shear stresses in the local coordinate system x_k-y_k, respectively. The origin of the local coordinate system is the center of the crack, and the x_k axis is parallel to the kth crack face. The boundaries of the finite domain (not including the crack surface) is B. Let the boundary conditions along B be L, which include stress boundary condition, displacement boundary condition and mixed boundary condition. The body forces are F^l ($l=1, 2$).

Decomposing problem P into an finite domain without crack, denoted as P_0, and N infinite domains with one crack, denoted as P_i ($i=1, 2, \ldots, N$), as shown in Fig. 1(b) and Fig. 1(c). For problem P_0, the unknown stresses on the location of the kth crack face in problem P is denoted by t_{0k}, For problem P_i, the unknown loads on the ith crack face is denoted by t_i.

Because the governing differential equations and the boundary conditions are the same in the above problems. According to the principle of superposition, the stress and displacement of problems P,P_0 and P_i satisfy the following relationships

$$\left.\begin{aligned} \sigma_x(x,y) &= \sigma_x^0(x,y) + \sum_{i=1}^{N} \sigma_x^i(x,y) \\ \sigma_y(x,y) &= \sigma_y^0(x,y) + \sum_{i=1}^{N} \sigma_y^i(x,y) \\ \tau_{xy}(x,y) &= \tau_{xy}^0(x,y) + \sum_{i=1}^{N} \tau_{xy}^i(x,y) \\ u(x,y) &= u^0(x,y) + \sum_{i=1}^{N} u^i(x,y) \\ v(x,y) &= v^0(x,y) + \sum_{i=1}^{N} v^i(x,y) \end{aligned}\right\} \tag{1}$$

where $\sigma_x(x,y)$, σ_y (x,y) and $\tau_{xy}(x,y)$ are stress in problem P and $u(x, y)$ and $v(x, y)$ are displacements in problem P. The superscript i denotes the value in problem P_i.

The boundary conditions on the position of Ω in P_0 are L_0 and the response at the boundary location of Ω in P_i are L_i ($i=1, 2, \ldots, N$). It should be noted that L and L_0 have the same boundary condition type, and the response type of L_i is also the same as the boundary condition type of L. Meanwhile, the loads on crack faces and boundary condition of Ω satisfy the following relationships

$$\begin{cases} L = L_0 + \sum_{i=1}^{N} L_i \\ T_k = t_{0k} + \sum_{i=1}^{N} \int_{-a_i}^{a_i} t_i C_{ik}(x_i, x_k)\mathrm{d}x_i (k = 1, 2, \ldots, N) \end{cases} \tag{2}$$

C_{ik} (x_i, x_k) denote the stress response of the kth crack face corresponding to the ith crack loads. In problem P_0, $N+1$ unknowns are involved (L_0 and t_{0k} ($k=1,2,\ldots,N$)). In problem P_i, $2N$ unknowns are involved (L_i