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Dispersion analysis for acoustic problems using the point interpolation method

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ABSTRACT

A typical meshfree point interpolation method (PIM) is presented to investigate the dispersion error in the numerical solutions of acoustic problems which is governed by the Helmholtz equation. It is well-known that those results from several numerical approaches, such as the finite element method (FEM) and several meshfree techniques, will suffer from the pollution effect, leading to the incorrect acoustic wave propagation for high wave numbers. The reason for this phenomenon is that the numerical solutions of wave number do not accord with the exact wave number, which is the so-called dispersion issue. In addition, to overcome the possible singularity issue in constructing the shape functions for the PIM with the polynomial basis functions (PBFs), the Gauss–Jordan elimination (GJE) technique is employed here. Several numerical examples concerning dispersion analysis and acoustic wave propagation are performed to verify the accuracy of results from the PIM. It is found that the PIM can reduce the dispersion error effectively and hence generate more accurate results than the FEM with the same set of nodes.

1. Introduction

There exists an intense demand for accurately investigating and predicting the acoustic propagation in order to achieve the requirements of special laws or regulations, such as depressing the ship cabin noise to improve passengers' travel experience. During the past decades, various numerical approaches have been developed to fulfill this object. Among these methods, the finite element method (FEM) [1] and the boundary element method (BEM) [2–5] have become the most prevalent tools to address acoustic problems governed by the well-known Helmholtz equation. The system matrices obtained from the classical BEM, however, are not sparse generally and not symmetrical when using the collocation method, resulting in the considerable difficulties in choosing the appropriate computing methods and the increasing computational cost. Recently, an innovative singular boundary method (SBM) have been well devised to tackle acoustic problems [6,7], which incorporates the origin intensity factor for the fundamental solution. This approach can effectively overcome some defects of the BEM but is still under development.

As a result, the classical FEM which is constructed with the sparse and symmetrical system matrices is always preferred by many researchers to approximate the solutions to the Helmholtz equation. The results obtained from the FEM are always affected by the pollution effect, which roots in the dispersion issue that the wave numbers from the numerical approaches do not accord with the exact wave numbers, especially for high wave numbers. With the aim to improve accuracy of the FEM for high wave numbers, the *h*-FEM and the *hp*-FEM are proposed [8,9] to relieve the dispersion error issue. These FEMs above incorporated with the high-quality mesh or the high-order polynomial basis would raise the computational cost greatly. Thus, several other improved FEMs have been developed for analyzing acoustic problems, such as the Galerkin least-square (GLS) FEM [10], the quasi-stabilized FEM (QSFEM) [11], the residual-based FEM [12]. Unfortunately, these methods still suffer from the limitations on either the insufficient effectiveness in depressing the dispersion error for the general two- and three-dimensional acoustic problems or being the relatively complicated in the formulation. Another approach called the partition of unity finite element method (PUFEM) in conjunction with a priori knowledge about finite element solution is proposed by Melenk and Babuška [13], showing a great reduction of the dispersion error compared with the GLS and the QSFEM. However, this methodology may produce the ill conditioned system matrices for the finite element model, leading to a tough operation for resolving such series of system equations [14]. Moreover, another alternative way is to redistribute the mass matrix for balancing the stiffness and the mass of a discretized model [15–20], which shows that the dispersion error can be reduced correspondingly.

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From several researchers' work [21–23], it is found that the essential root of the dispersion effect in solving acoustic problems may derive from the "overly-stiff" property of system stiffness matrix computed by several numerical approaches above. Hence, it is very crucial to find certain techniques that can properly "soften" the numerical model. Recently, Liu and his research group have proposed a series of S-FEMs [24–27], which combines the standard FEM with the gradient smoothing technique (GST) [28] on the basis of G space theory [29–31], to tackle acoustic problems. The GST could properly relieve the excessive stiffness of the finite element model and hence render a proper numerical model whose system stiffness is closer to the exact model. Therefore, these S-FEMs [21–23,32–39] can provide the superior advantages to reduce the dispersion error and relatively ensure the accuracy of results for solving acoustic problems. Nevertheless, these mesh-based FEMs cannot easily be applicable to adaptive analysis due to the presence of mesh grids in the problem domain. In comparison with the procedure for the FEM, the major superiority of meshfree methods [40,41] over the FEM is that the mesh generation is not necessarily required in the formulation of shape functions, implying that it can get rid of manual operation on the pre-processor task (such as the mesh generation), and be automatic to tune the mesh density for interpolation by programming. Hence, meshfree methods could have a good adaptability for adaptive analysis.

Meshfree methods are the relatively novel approaches to investigate acoustic problems. A typical meshfree method, called as the element-free Galerkin method (EFGM) [42], was employed to analyze the pollution effect for acoustic problems by Bouillard and Suleau [43]. It is shown that the EFGM can eliminate the pollution error greatly in contrast to the FEM for high wave numbers and hence have higher order of accuracy than the FEM. However, the EFGM using the moving least square method (MLS) has a tricky drawback that the essential (Dirichlet) boundary condition is relatively difficult to be implemented due to the lack of Kronecker Delta function property.

Recently, a new meshfree method with the desirable nature of Kronecker Delta function, which is named as the point interpolation method (PIM), is proposed by Liu [44] and Liu and Gu [45], it has already shown a good adaptability to the field of solid mechanics [46–50]. Compared with the EFGM, this methodology can handle with the essential (Dirichlet) boundary condition as simple as the operation used in the FEM. The other major superiorities of this polynomial PIM are simple in the formulation of the shape functions and of high accuracy [45], but the possible singularity issue of the moment matrix is a key drawback which restrains the development of the PIM with the polynomial basis functions (PBFs). Therefore, a simple and effective Gauss-Jordan elimination (GJE) technique is utilized to overcome this issue in this paper, which will be detailed in the following section. Another way to overcome this issue is to replace the PBFs with the radial basis functions (RBFs), which is thereby called as the radial point interpolation method (RPIM) [46]. But the approximations from the RPIM might fail to be convergent in terms of the refinement of mesh owing to the lack of the ability to reproduce the linear field [51]. Hence, it is usually recommended that the RPIM should be augmented with the linear PBFs [45,51].

However, as far as the authors' knowledge is concerned, the PIM with the PBFs has seldom been used to handle the Helmholtz problems, though the good features of this meshfree technique has already been demonstrated for the solid mechanics problems. This paper mainly focuses on extending the application of the PIM with the PBFs from solid mechanics discipline to acoustic problems. Due to the good performance of the polynomial PIM in solving solid mechanics problems, it is expected that the PIM with the PBFs will also behave better in controlling the dispersion error and could provide much more accurate numerical solutions in analyzing acoustic problems than the conventional FEM with the same set of field nodes.

This work is organized as the following. In Section 2, we briefly introduced the derivation of the corresponding governing equations for acoustic problems. The formulation of the PIM is conducted with the GJE technique for overcoming the singularity issue in Section 3. In Section 4, the knowledge of dispersion analysis is illustrated with the detailed formula derivations. Several numerical examples are presented in the Section 5. Finally, several conclusions can be drawn regarding the advantages of the PIM with the PBFs in Section 6.

2. Acoustic problems governed by the Helmholtz equation

2.1. Governing equations for acoustic problems

Acoustic wave propagation in the acoustic medium is governed by the wave equation expressed by

$$\nabla^2 P - \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} = 0 \tag{1}$$

where t denotes the time variable, c represents the acoustic velocity in the acoustic medium and P is the harmonic acoustic pressure described as

$$P(\mathbf{x}, t) = p(\mathbf{x})e^{j\omega t} \tag{2}$$

in which ω is the angular frequency and $p(\mathbf{x})$ signifies the complex acoustic pressure.

By substituting Eq. (2) into Eq. (1), the well-known Helmholtz equation is derived as

$$\nabla^2 p + k^2 p = 0 \tag{3}$$

where k is the wave number defined by $k = \omega/c$.

Providing that the domain Ω of the acoustic problem is enclosed by the boundary Γ consisting of the segments of three different boundary conditions, such that

$$\Gamma = \Gamma_D \cup \Gamma_N \cup \Gamma_R \tag{4}$$

where Γ_D denotes the Dirichlet boundary expressed as

$$p = \bar{p} \tag{5}$$

Γ_N denotes the Neumann boundary indicated as

$$\frac{\partial p}{\partial \mathbf{n}} = -j\rho\omega\bar{v}_n \tag{6}$$

and Γ_R denotes the Robin boundary defined as

$$\frac{\partial p}{\partial \mathbf{n}} = -j\rho\omega A_n p \tag{7}$$

in which $\mathbf{n}$ represents the outward normal vector of certain boundary, $\bar{v}_n$ stands for the normal velocity along the boundary, A_n is the admittance coefficient and ρ is the density of the acoustic medium.

For acoustic problems, the gradient of acoustic pressure p is related to the particle velocity v of acoustic wave by the equation of motion, which can be expressed by

$$\nabla p + j\rho\omega v = 0 \tag{8}$$

2.2. Weak form for acoustic problems

Here, the weighted residual method is employed to address the Helmholtz equation, which yields an integration over the entire domain Ω as follow

$$-\int_{\Omega} w(\nabla^2 p + k^2 p)\mathrm{d}\Omega = 0 \tag{9}$$

in which w is the test function.

Using the Green's theorem, Eq. (9) leads to

$$\int_{\Omega} \nabla w \cdot \nabla p\mathrm{d}\Omega - k^2\int_{\Omega} w \cdot p\mathrm{d}\Omega - \int_{\Gamma} w(\nabla p \cdot \mathbf{n})\mathrm{d}\Gamma = 0 \tag{10}$$

By substituting the Neumann boundary condition and the Robin boundary condition into Eq. (10), the weak form of the Helmholtz equation can be expressed as